STANFORD RESEARCH INSTITUTE
Menlo Park, California 94025 · U.S.A.

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CONSTRAINED MATRIX GAMES IN FLEET DEFENSE PROBLEMS--
THE BASIC MODEL.

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By: William H. Frye

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NAVAL ANALYSIS PROGRAMS (CODE 431)
OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
ARLINGTON, VIRGINIA 22217

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I. INTRODUCTION

The overall objective of this project is to apply game theory methods to realistic Fleet Defense problems in order to obtain answers in certain areas where no satisfactory answers now exist. This document covers Phase 1 of the larger effort continuing the approach described in a pilot study entitled "An Approach To A Game Theoretic Treatment of Fleet Defense" hereafter referred to as Reference 1.

There are two classes of problems we are concerned with: Fleet Defense Planning Problems and Force Resource Tradeoff Problems. In Fleet Defense Planning problems the composition of both sides is fixed, each side is uncertain about what the other side has and how it will use what it has, and the planner wants to know how best to use his forces when opposed by an intelligent, responsive enemy. The principal study outputs of value in this class of problems are the optimal decisions (equivalently, the optimal game-theoretic strategies). Force Resource Tradeoff problems, the second class, presume solutions to the first class are available and generalize the tactical problem by allowing the composition of one or both sides to change, constrained by considerations such as total cost. As in the Fleet Defense Planning problems, tactical decisions are optimized; the difference is that resources are optimized as well. In short, the resource tradeoff problems jointly optimize resource decisions and tactical decisions. Clearly the Force Resources Tradeoff problems are the more difficult of the two classes, and in some respects the more important. However, this project (Phases 1 and 2) is concerned solely with Fleet Defense Planning problems and accordingly we will not mention Force Resource Tradeoff problems again in this technical note.

Our approach, ~~in brief~~, is to take a Fleet Defense scenario involving the protection of a Carrier Task Force against enemy air attack, formulate the two-sided tactical problems in game-theoretic terms under uncertainty, and solve the problem by CMG methods in order to answer the scenario's basic active-passive questions and aircraft and missile employment questions.

II. THE FLEET DEFENSE SCENARIO

This section has a two-layered description of the Fleet Defense scenario. The first is a summary description adapted from Reference 1, updating and enriching the scenario as specified in Task 1 of the proposal. The second description is independent of the first and more specific; sufficient information is provided to serve as a specification for a Monte Carlo simulation program.

The second description is to be regarded as defining the tactical situation and the Blue and Red tactical problems for the entire project. Models given in later sections will not necessarily be as general as the description, however.

In order that no interested readers should be left behind, the descriptions are in narrative form, augmented by some simple geometric diagrams. It is important that Naval officers as well as operations and systems analysts should be able to read the problem statement, at the minimum, in order to appreciate the nature of this research undertaking. This is not to say that the nontechnical reader will be left behind in later sections, every attempt will be made to keep interested readers aboard. (The single basic technical fact that many will have to accept is that well-developed methods exist for solving zero-sum, two-person (ZSTP) games once the game is cast in the proper format. This format, called the CMG game matrix format, is developed in a later section called "A Sequence of Games."

The Event-Flow diagram for the scenario appears in Appendix C.

A. Summary of the Scenario

A Blue Carrier Task Force (CTF) is in transit, passing within range of a Red land base capable of supporting attack aircraft (bombers). Blue wants to complete the transit without loss or damage to ships or aircraft,

while Red wants to sink or at least damage the CV without loss of bombers. A Red satellite may already have provided detection of the CV, unbeknownst to Blue.

The CV may choose to deploy Combat Air Patrol (CAP) aircraft for search, investigation, and intercept. (By CAP is meant a combination of fighter and Airborne Early Warning (AEW) aircraft. There are Deck Launched Interceptors in the problem as well, and the tradeoff between CAP and no-CAP defenses is included in the game.) The AEW aircraft may search either actively or passively, and may switch from passive to active at various points in the game. AEW aircraft have detection capability against low-flying bombers while the CV's radar does not.

The CV has to decide initially whether to be in EMCON or not, and if EMCON is selected, the CV has to decide later (based on tactical events) when and whether to break EMCON. (EMCON, or Emission Control, is used here to mean that the CV radar is not used and Blue communications are minimal.)

A Red reconnaissance aircraft ("Recon") flies along a prescribed search path, searching either actively or passively for the Blue force. Recon may detect either the CV or one of the CAP aircraft, and, upon localizing the CV sufficiently well, calls in the bombers for an attack on the CV. The Bombers then begin flying out towards the CV, using position information provided by Recon. (Position information is encoded in the form of a time-varying probability area whose size and shape is responsive to detection events. At the start of the game the area is defined by intelligence estimates, a priori considerations, and satellite detectability considerations.) Approach and attack planning must therefore be done probabilistically, with the attendant possibilities of making mistakes.

Since some time is required for the flyout and since in any case the position information is imperfect, the attack aircraft usually need further assistance in locating the CV. Knowing this, the Blue force may attempt

to destroy the Recon before the Bombers launch their weapons (antiship cruise missiles, or ASCM) at the CV. On some occasions the recon may face, with high probability, destruction by Blue without contributing commensurately to the likelihood of damaging the carrier. Therefore, at appropriate times, the Recon is allowed to flee from the area and withdraw his support from the Bombers. Similarly, the Bombers may be unnecessarily endangered by Blue defenses or have inadequate fuel to return home following mission completion; opportunities are permitted the Bombers to break off the mission and return home without having damaged the CV.

Blue AEW aircraft are in the meantime searching, and may detect the raid even at low altitude. Detection may permit interception by CAP or Deck Launched Interceptors (DLI) to counter the raid. Blue may also elect to orbit an interceptor over the CV in anticipation of the attack.

The Bombers carry one of two types of ASCM, each has different characteristics. Blue does not know the ASCM type the Bombers have, but does know the probability of each type. Bombers may elect to make their final approach to the CV from any direction after considering fuel requirements and the additional time such an approach may take. The generic bomber approach profile starts with a cruise phase at cruise altitude followed by a dive to a low altitude to avoid possible detection by the CV's radar. Whether the Bombers need to climb at the end of this low-altitude phase is largely a function of the size of the probability area: if the probability area is small (and the ASCM design permits) the ASCM can be launched at low altitude. However, if the probability area is large (as when Recon has been shot down) the Bombers will have to get better information by climbing to a search altitude and searching for the CV on radar. The Bombers are vulnerable to detection by the CV radars and attack by Blue interceptors during this search phase and at all times thereafter. Interceptors may choose to pursue bombers even after ASCM launch has occurred, and the bombers must plan for successful egress as well as approach and

and attack. Bombers may make various mistakes during the critical climb-to-search-altitude maneuver. If they climb too soon (beginning search at too great a range from the CV) they may be vulnerable for a longer period than necessary to detection and ultimately destruction by the Blue defenses. On the other hand, if they climb too late they may be closer to the CV than necessary at the onset of search, and again be more vulnerable than necessary. In some cases they may overshoot the CV target altogether, depending upon the probability area and search plan. Fuel problems may also develop for the Bombers, especially when the CV turns out to be at a significantly greater range than the Bomber had thought.

Blue's final round of defense is the autonomous Point Defense Missile System (PDMS) which employs a weapon resembling Sea Sparrow. The PDMS sensor detects objects with a higher probability when alerted by an earlier AEW or CV detection, but otherwise the PDMS has minimal interaction with the CV radar/interceptor system.

Numerous decisions have to be made by Blue near the end of the game concerning firing doctrine, missile and interceptor allocation, and firing range. The game ends in one of several possible outcomes. Basically what is important about the outcomes is the number of hits on the CV, Bomber losses due to Blue action or running out of fuel, losses of Blue aircraft, and loss of Recon.

Decisions made by Blue and Red are concisely given in section II-E. We merely mention here that there are several decisions concerning when to go active, which enables analysis of active-passive questions. Also, many countermeasure considerations may be built into the weapons effectiveness models for analysis and study. Communications can be intercepted by the enemy, making the decision as to whether to communicate or not important in some contexts. Still other decisions concern interceptor and weapon allocations, these are usually of the form found in routine operations analysis studies.

B. The Fleet Defense Scenario In More Detail

This section describes the tactical situation and tactical problems for Blue and Red in sufficient detail to act as a specification for a Monte Carlo simulation model. However, the language of the operational navy is employed as much as possible.

Because of the importance of uncertainty, it is necessary to make clear just what information each side has throughout the game. Neither knows the other's decisions, this being an essential feature of game theory. To specify knowledge about other aspects of the problem in a straightforward, positive way results in very cumbersome text. For example, there is a Red airfield in the problem. It is also the case that Blue knows:

1. That there is a red airfield
2. The location of the airfield
3. That Red knows that Blue knows (1) and (2)*.

The short-cut way to describe uncertainty is by negation: each side knows all it needs to know except that which is specifically excluded. As an example, it will later turn out that although Blue knows the type of Red Bomber, and numbers of Red Bombers, he does not know the kind of ASCM the bombers carry--in the terminology of this project Blue is uncertain about the kind of ASCM.

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*This third point, which suggests infinite iteration (Blue knows that Red knows that Blue knows,...) is not a trivial matter. According to Harsanyi (Ref 2) it was precisely this apparent need for indefinite iteration that had blocked progress in handling uncertainty in games until his theory was developed. Fortunately we do not have to do the iterations, for the theory does not require them.

1. <u>Elements of the problem</u>

The physical elements entering into the problem are:

For Blue--

- ships of a Carrier Task Force (CTF)
- interceptors aboard the CV
- air-to-air missiles (AAM) for these interceptors
- a Point Defense Missile System (PDMS) on the CV, and its missiles
- radars for the ships
- ESM equipment for the ships
- communications equipment for the ships
- a command/control system
- intelligence regarding Red forces

For Red:

- an airfield
- reconnaissance aircraft
- radar, ESM, and communications equipment for the reconnaissance aircraft
- a surveillance satellite
- bombers
- antiship cruise missiles carried by the bombers
- communications equipment and radar for the bombers
- a command/control system
- intelligence regarding Blue forces

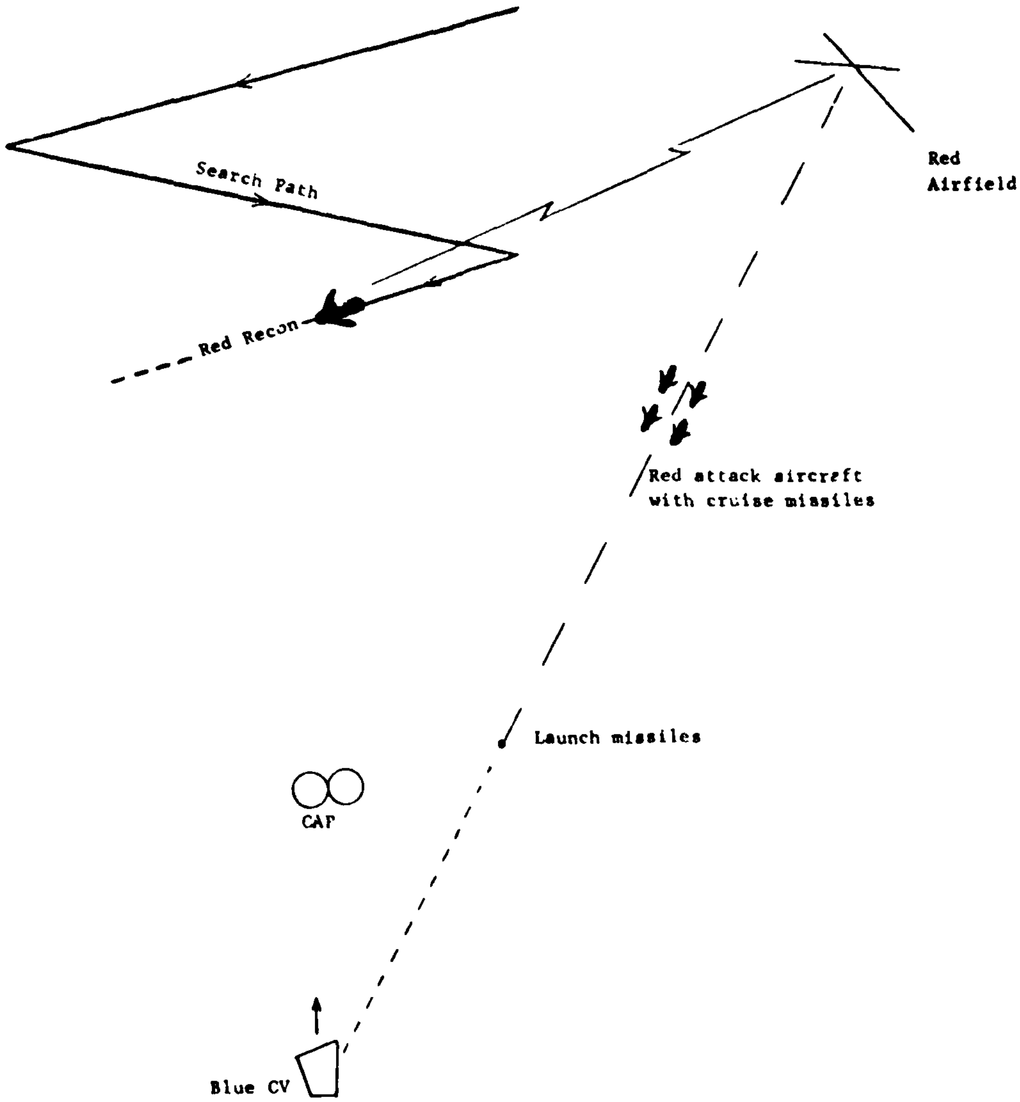
2. <u>Getting the problem started</u>

A Blue CTF is in transit to an unspecified destination within range of Bombers at a Red airfield located on a nearby land mass. Red is expecting the CTF transit and therefore has reconnaissance aircraft

("Recon") searching for the Blue force. Red also has Bombers on the alert at the airfield to run out and attack the carrier upon command when adequate position information becomes available from satellite or recon surveillance.

Red may or may not have satellite surveillance, and there may or may not have been satellite detection of the CV by the game's starting time at t = 0. These two possibilities are combined into one--there is either CV detection by satellite at t = 0 or there is not, and such detection occurs with a probability known to Blue. In any given play of the game Red knows whether or not there is detection. He also knows that Blue knows the probability of detection. This structure will occur again and again in this model--one side will know whether an event has occurred or not, and the other will only know the probability of the event. It is always assumed that the probability is the true probability of occurrence. Satellite detection is not dynamic, which means that if it has not occurred at t = 0 it will not occur later in the problem. This assumption, and many others like it, are made simply to help bound the problem. That is, the methodology will handle the dynamic case if we choose to formulate the model that way, but since detection of the CV will be dynamic in other ways it seems to be an unnecessary complication to also include dynamic satellite surveillance.

Figure 1 shows the initial geometry of the problem for the particular decisions indicated. The Recon follows a single search path, starting from a random point selected from a prescribed portion of this path. Blue knows the portion of the path from which the initial Recon position is selected and the probability law of selection, but does not know the point. In contrast to Red's random start, Blue makes a decision: he chooses the x value (effectively the closest point of approach to the airfield) of his straight-line track. Since this is a Blue decision, by the discussion earlier we know that Red does not know the value of x.



Notes: 1) Recon detects CV or CAP, calls in attack aircraft.
2) If TF detects Recon, may try to intercept before detection of CV or launch of cruise missiles.

FIGURE 1 A SIMPLE FLEET DEFENSE PROBLEM

However, Red does know the possible values of x from which Blue can choose. This kind of assumption is another recurring feature of the model--whenever one side makes a decision, the other side knows the possible values or levels of that decision. I.e., the other side knows the candidates for decision.

This is not yet enough specification to get the problem started. There are some kinematic considerations: all speeds are constants known to both sides, there are no delays, slowdowns due to turns, etc., unless otherwise specified. When speed changes are involved they are instantaneous, implying that speeds have to be averaged in some sense. At the problem level we are considering these are presently thought to be reasonable assumptions. However, events involving crucial timing will have to be watched carefully. An example is the launch of an Interceptor to intercept a Bomber--the timing may determine whether the Bomber is hit before ASCM launch or after, and which of these occurs is often of importance.

Other initial conditions are determined by Blue and Red decisions. Taking Blue first, he has to decide whether or not to have CAP, and if so which of the specified stations will it be? (There are a small number of possible stations, not more than ten. Red knows these are the possibilities, since the choice of station is a decision.) The term CAP implies both interceptors and AEW aircraft. A CAP station consists of an (x,y) track relative to the CV, most commonly a race-track pattern. Some randomization is required for obtaining initial positions, and if it matters the direction of flight will be clockwise from some specified point on the track. For definiteness it will also be assumed that an Interceptor and an AEW aircraft maintain some definite relationship to each other, one aircraft may be a half cycle from the other, say.

Further Blue decisions concern sensor use at the beginning of the game. For the CV, is the radar to be on, or off? Is the AEW aircraft

to be active, or passive? i.e., should its radar be on or off?

For Red, it has to be decided whether the Recon's radar should be on, or off. Also, Red may decide to send the Bombers out to attack the CV based on CV position information available at the start of the problem. If they are sent out there are target approach decisions to be detailed later, obviously a course and altitude are needed as a bare minimum.

The above is sufficient to get the problem started. In simulation terminology, the simulation can be initialized. Everyone knows where to go and what to do at the outset.

3. Objectives for Blue and Red

The players cannot make decisions intelligently without having an objective. What will happen during a game is that some series of tactical evolutions will occur, terminating in an event which ends the game. Each such series of evolutions can be considered a path through an event-space, with each path having a definite ending or end-state. This end is also called an outcome. The utility theory and decision analysis viewpoint is adopted here with respect to outcomes: each outcome will have associated with it a single number, and this number will be an input parameter. This number represents the value of the outcome (in ordinary language) or the utility of the outcome (in utility theory and decision analysis language).

A crucial assumption about this game is that it is zero-sum, which means that Blue is directly opposed to Red and conversely. In particular Blue wants the carrier to survive without being hit by cruise missiles and Red (with equal "intensity") wants the carrier not to survive without a hit. Similarly Blue values Red's Bomber as much as Red does, the difference being that Red wants the Bomber to return to the

airfield undamaged and Blue wants to splash it, or have it run out of fuel before returning to the airfield.

This payoff or effectiveness concept cannot be made precise without numbers or without taking probabilistic fluctuations into account. Even for fixed, deterministic decisions the game outcome will ~~be random.~~ Why?
Hence the game payoff can be taken to be the average utility of an outcome, where the averaging takes place over all random elements of the problem, including any randomized decisions that may be made. It is an old question in utility theory and other areas as to the appropriateness of an average value measure in a game that is to be played just one time. Suffice it to say here that there are good arguments on both sides of the issue. We simply assume this averaging concept is valid for defining payoff. Although the term MOE is not needed in the sequel, it can be remarked that the payoff as defined here corresponds to what is usually called a measure of effectiveness (MOE) in operations and systems analysis.

Since one side is directly opposed to the other, we may think of the utilities (one associated with each outcome) as being chosen by Blue. Blue prefers higher utilities to lower ones, and, for fixed Red decisions, will try to maximize the payoff (the average utility) as his overall objective. Because of the zero-sum assumption, Red merely tries to oppose Blue--this he does by minimizing (for fixed Blue decisions) the same payoff. (The reason for the stipulation of "fixed Blue (or Red) decisions" is that game-theoretic maximization and minimization is two-sided in that it is recognized by each side that he will be opposed by his opponent.)

In summary: we assume a single number representing the utility of an outcome is associated by input with the end-point of each possible path representing a possible way the game may evolve. Blue wants to maximize the average value of this utility and Red wants to minimize it (zero-sum property); the average utility is also called the payoff.

What is actually done here is to use a payoff (an element in the payoff matrix) which is itself an average value, where the average is taken over all nondecision elements of the problem which are random. That is, if one were to actually form the huge payoff matrix required in the ordinary basic ZSTP game theory framework, each payoff element would be an average value obtained (say) by replicating a simulation model many times with fixed decisions for each side. It is the need for this replication, in conjunction with uncertainty and the very large game matrix, that defeats the straightforward application of zero-sum, two-person game theory to this problem.

Some further discussion is needed of the utility concept being used. First, it has to be emphasized that the utilities associated with outcomes are <u>inputs</u> to the problem. Because there are so many possible paths there is the inevitable need to aggregate the terminal states to reduce the effort involved in determining utilities. Aggregation will not create any difficulty in this problem.

More difficult is the assessment of the value of outcomes that involve losses to both sides. Blue will have to decide, for example, whether he would rather splash the Bomber and have his own interceptor lost, or fail to splash the Bomber (who returns home for an attack on another day) while saving his own interceptor. Similarly a hit on the CV has to be weighed against loss of a Blue interceptor or AEW aircraft, and so forth. All truly different outcomes have to be ranked and assigned a value, and this will often involve what are in real life painful choices.

Having made these assignments, however, a lot of decision problems we are used to seeing handled in other ways are automatically taken care of. An example of this occurs on pages 28 & 29 where it has to be decided how long the Recon should continue to support the Bomber during the Bomber's run-out to attack the CV.

4. The Probability Area Concept

A key element used in attack planning is a probability area (PA). The PA idea is roughly the same as the Sosus Probability Area (SPA) in Sosus contexts: instead of specifying a point as the position, an entire area is specified. The target is assumed to be in this area with some probability and its position conditionally distributed within the area by some given probability distribution. For simplicity we will assume here that (1) the probability area always contains the CV target, and (2) the probability distribution of the CV position within the area is uniform. (Actually we will approximate the area by a small number of points, each of them having the same probability.) The probability area is dynamic, changing with time and events. A detection in general shrinks the probability area, and loss of contact allows it to grow. Active Recon detections have smaller areas than passive ones, and have a different shape. When there is no detection at all the probability area is a priori or perhaps intelligence-based and is therefore likely to be quite large. At the other extreme and active Recon detection may reduce the area to a single point. Conceptually target classification may also be included in this framework by removing the uniform distribution assumption. If the CV is known to be one of four ships (say), then the probability area may be approximated by four points, one at each ship location, with probabilities of the ship being the CV associated with the points.

5. The Attack Concept for Red

Red's tactical planning problem is familiar: find the CV and send Bombers out to sink it with antiship cruise missiles. Red will try to do this in such a way as to minimize his own losses, i.e., lose neither the Recon nor any Bombers.

Finding the CV may be no problem, for satellite surveillance may already have provided position information sufficient for starting

the attack run-out. However, we assume that Red has decided that a reconnaissance aircraft will be used--if its detection information is not needed it can act as a decoy to hopefully nullify a Blue interceptor.

Consider the case where there is no satellite detection. The attack concept is to have the Recon detect the CV (or some other element of the Blue force) and relay position information to Red command/control or directly to the Bomber to enable the Bomber to make a successful attack.

In the simplest case the Recon detects and at some time transmits the detection information (i.e., the probability area) to Red command/control, remaining in the area to assist the Bomber. Sometime after the receipt of the message Red command/control (or simply Red) decides to have the Bombers take off, beginning their run-out to attack the CV. The Bomber runs out to the area based on decision parameters involving both the approach path (the ground track) and a profile (altitude). One possibility, which is anticipated to be often the form of the optimal answer, is as follows: the Bombers run out at cruise altitude in the general direction of the middle of the probability area, run around the force through a selected direction while maintaining range such that detection by the CV is impossible even if its radar is on, drop down to a low altitude to close the expected CV position, climb and search for the CV, detect the CV, launch the ASCMs, drop down to low altitude again, open the CV until out of danger, and finally climb back to cruise altitude and return home.

There are many variations on this attack form, and several things to be considered in deciding on parameters for the attack. Cruise altitude is used as much as possible to save fuel, and the dropdown point is based on fuel and susceptibility-to-Blue-detection. (Detectability in turn relates to the probability area, for the profile usually

cannot be chosen under the assumption that the CV position is exactly known.) The climb made by the Bomber to search for the CV is made also based on the probability area. If the CV position is perfectly known it is not necessary for the Bomber to detect, and he can often launch the ASCM at low altitude. Otherwise he must climb to search, which he generally does as late as possible to avoid exposure to the CV's radar and air defense. How high to climb and how to search when at this altitude have to be decided using the probability area and other factors.

At various instants of time during the engagement the Recon will be allowed to flee if it is warranted. (This means that the decision to flee will be permitted as a candidate decision and that the methodology's algorithm will determine whether fleeing is warranted.) Similarly the Bombers will have opportunities to break off their mission and return home without attacking the CV. An example where this is a reasonable Red decision may occur when a probability area changes in such a way during run-out that it is now much more probable that the CV is at a greater distance from the airfield than was thought to be the case earlier. By continuing his present attack plan, or even by shifting this plan to one requiring less fuel, it may work out that the probability of the Bombers running out of fuel is quite high. They may therefore decide to break off the mission and come home.

6. The Defense Concept for Blue

Blue knows the surveillance and attack concept and all the surveillance and attack parameters except whether or not a satellite has already detected and the type of ASCM the Bombers will carry. Blue does not know Red's tactical decisions. Neither does he know tactical events such as whether or when the Recon detects the CV, whether the Recon is being used as a decoy, when the Bombers begin running out to the CV, and where they will drop down to low altitude to avoid detection.

Corresponding to the probability area concept for the attacker is the concept of probabilities of these probability areas for the defense. That is, Blue will be assumed to "know" in a probabilistic manner what probability area Red has. Furthermore, as required by the methodology, Red is assumed to know the probabilities that Blue has. All this is discussed further and justified later by referring to a simulation model which can be developed that is satisfactory to both sides for their analysis.

Special cases of the probabilities of PAs should be pointed out: if all Blue's probability is on one PA, and this PA in turn consists of a single point, then Red knows precisely where the Blue CV is and Blue knows that Red knows and Red knows that Blue knows that Red knows. As a second example, Blue may have all the probability on one PA, thus they both have the same probability distribution for the CV. This situation could result from Blue intercepting a Red message containing the probability area, providing Red knew that Blue had intercepted it.

We consider now Blue decisions. Blue has several active-passive kinds of decision, which ultimately come down to radeoffs between detecting and being detected. There are two basic ways for Blue to attempt the transit: 1) go active all the way from the start of the problem and 2) start passive and remain passive for some period, going active only when warranted. (Active means essentially "radar on" and communicate freely, passive means "radar off" and communicate minimally or not at all.)

Certain other Blue decisions have been discussed under "starting the problem." These relate primarily to CAP and AEW. There may or may not be CAP interceptors and AEW, and if there are there will be a patrol path chosen for them. Initial fuel state and positioning assumptions involve randomization, not decisions.

Blue decisions are often determined based on the probabilities of the PAs, he will tend to go active when the overall composite probability

density determined by these probabilities and PAs is sufficiently narrow. That is, Blue will tend to go active as it is more likely that he has been detected and localized already.

Blue has to decide whether to replenish CAP when the interceptor becomes fuel-limited. In some situations Blue considers whether to launch an interceptor to orbit the CV in anticipation of the raid arrival. Blue must always consider detectability of elements of his force (AEW and interceptors on CAP, as well as the CV) by Red ESM, in particular the communications involved with orbiting an interceptor are assumed detectable by the Recon with a given probability and by the raiders by another given probability.

Allocation decisions arise for Blue: should the Recon be intercepted upon its detection by Blue? If so, should CAP perform the intercept? Or DLI? When the raid is detected the same kinds of questions arise: should CAP do the intercept? Or DLI? Or, on some occasions at least, should both? Other allocation decisions for Blue have to do with shooting down the Bomber vs. shooting down the ASCM. Getting the bomber before launch gets the missile too, but the early launch may have a lower hit probability against the Bomber. A certain number of salvo versus shoot-look-shoot questions may also be worked into the model as quantification and computerization proceeds and the magnitude of the increase in problem size is easier to assess than at present.

Another active-passive question concerns use of the interceptor's AI radar. If the Recon (or Bomber) is passive there is an Interceptor advantage to approaching the Bomber passively, turning on the airborne intercept (AI) radar as late as possible before launching his air-to-air missile. The radar turn-on is withheld to deny or at least partially nullify the Recon (or Bomber) the use of countermeasures. Thus there are two intercept cases, one for an unalerted target and the other for an alerted target.

Interceptors (from CAP or from the deck) are the first round of defense for the CV. The second round is the CV's Point Defense Missile System, and some decisions for Blue are induced by its interface with the interceptors. In cases where either an interceptor or a Sea Sparrow (say) controlled by the PDMS can be brought to bear against a Bomber or (more likely) an ASCM, a decision is needed as to who should pursue the target. Further details of this must await detailed modelling of this aspect of the problem, one expects decisions to be based on kill probabilities for the two missiles and the number of ASCM targets in the immediate area, at the minimum.

7. Further Scenario Details

The following information is provided to further specify the tactical model and a simulation program based on this model. Some of the points made earlier are duplicated.

a. The Recon search path (in x,y coordinates) is given and is known by both sides. Recon's initial position is randomly drawn over a given segment of this path. The segment is also known to Blue. Randomization is used only to prevent any undesirable correlation of initial positions for Red and Blue.

b. The CV's path is a straight line known also to Red. Thus Red in a sense "knows" the CV position at all times, however, he cannot act on this information but must use his sensors to obtain CV information.

c. Search models (both active and passive) are of the form: range and altitude determine a probability of detection per unit time.

d. When the Recon detects the CV actively, he effectively orbits and maintains the range at the time of detection. When CAP is detected, the Recon continues down the prescribed search path. When

the Recon detects the CV passively, he closes range down the bearing to the CV.

e. A CAP station is defined by a single index number, and associated with it is the geometry of the track. Red knows the candidate CAP stations as well as Blue. Some randomization is involved in placing an Interceptor at the CAP station and giving it an appropriate fuel state. Red knows the randomization rules but not the outcome.

f. Recon is given opportunities to flee

- following initial detection and relay of CV position to Red command control
- when an Interceptor is detected being launched from the deck for any purpose
- when an Interceptor is detected starting out to intercept the Recon
- same as above, but at the latest possible time for Recon safety.

g. The CV has a radar which may be on or off and ESM which is always available. ESM can detect, in particular, the Recon-to-Red command message concerning CV detection. This detection occurs with a probability that is known to both sides. ESM cannot detect the Recon's radar if the CV's radar is on.

h. Recon has to decide when to transmit a detection to the Red command control. He knows the transmission may be detected and exploited by Blue. Therefore he does not want to transmit too early. On the other hand, if Recon waits too long Blue may detect him and may even have an attack underway on him, in addition to giving Blue a chance to alert his defense and to put an interceptor in orbit if desired. Recon is given an opportunity to transmit the detection message any time there is a stimulus to change the probability area. It is thus possible for Recon to get a passive detection with a large PA, switch to active, detect the

CV actively on radar, and then transmit the message to Red command control.

i. The CV may initially be active (radar on) or passive, and upon changing from passive to active no reversal is possible, i.e., once active, always active holds for the CV. Opportunities for changing from passive to active are:

- upon a passive detection of the Recon (intercepting the message to Red command control)
- at the time of passive detection plus a delay: several delays may be considered, precalculated based on hypothesized actions and/or state of Red. For example, CV may go active when he estimates there is a 10% probability of the Bomber being detected if the Bomber is at high altitude. "Probability" here can be very broad, including estimates of Red's strategies. We can think of the candidate times as precalculated, however.

j. Blue makes an initial decision as to whether to have CAP, with randomization as discussed in section II B.5. An interceptor may have to be launched to replace the CAP, depending upon how the initial fuel state random draw came out. This launch and coordination with the returning aircraft requires communications which are interceptible by Recon. Again, a probability known to both sides is assumed given.

Launches may be made by Blue on any subset of the following set of occasions:

- replace an exhausted CAP, if CAP has been selected
- upon detection of Recon, to either orbit (over the CV), or for intercepting the Recon
- upon detection of the raid by the CV or by an AEW aircraft in order to intercept the raid.

k. Aircraft may leave CAP on the following occasions:

- when fuel state is low, returning to the CV. Such an aircraft is unavailable for intercept.

- to intercept the Recon at the time the Recon is detected. A bearing on the Recon is sufficient information.

- to intercept the raid when it is detected.

Interceptors can approach targets for intercept in more than one way. If good position/velocity information is available from the CV, the interceptor can remain passive until within range of its own air-to-air missile, go active briefly, and get the launch off in such a way as to give the Red target aircraft little time for countermeasures. Result: enhanced probability of kill of target. If information available was poorer, however, Interceptor will have to go active earlier, giving the Red aircraft more time for countermeasures and hence lower probability of kill. Thus active detections by the CV are of more value to Blue than passive ones.

l. When the raid is detected doing its final pop-up search, an interceptor going after the raid is forced to be active. This alerts the raider, and results in a lower probability of kill than the unalerted case.

m. It is better for Blue to detect the raider at long range than at short range, in general. This is because of the increased alertment of Blue and the extra time that can be used to iron out communications difficulties, fix minor equipment problems, etc. These considerations are reflected in a deterministic relationship of delay time and time of detection (meaning the length of time from detection to the time the ASCM is expected to impact). Following this delay time an interceptor is assumed to fly at constant speed. It's air-to-air missile also flies

at constant speed, and is launched at a chosen range not greater than maximum range or the range when launch is first possible.

n. There are two possible ASCMs for the Bomber, Blue knows the type only probabilistically. The types differ in their radar cross-section paramater and in their profiles. Each missile is limited by a maximum range and can be launched from specified altitude bands. Once launched their profiles differ, influencing detectability, vulnerability to Blue and their hit probability against the CV. Warheads may also differ, in which case the utility values will be different for hits by the two types of missiles.

o. The Point Defense Missile System (PDMS) aboard the CV has its own sensors, independent of the CV's radar. Detection by this system is of the same form as other sensor models. However, alertment by Blue radar detection enhances PDMS detection. There are thus two separate curves for PDMS detection: 1) unalerted and 2) alerted by detecting.

Given detection, the probability of the PDMS's missile (say a Sea Sparrow) killing the missile is dependent upon range at detection. We assume that the Bomber may also get close enough to the CV to be in danger from the Sea Sparrow (probably by miscalculation--recall the probability area concept). Interceptors do not get in that close (by assumption), and Blue's missiles do not interfere with each other. When more than one missile shot homes in on a given target the kill attempts are assumed statistically in the independent kill evaluation.

p. The Red planning for the approach to the CV involves such realistic consierations as fuel limits and inability to find the CV. If at all possible, the Bomber would like to fly low throughout the entire surveillance region of Blue in order to avoid detection. Flying low takes extra fuel, and inhibits the ability of the Red Bomber to detect the CV on its own--these are the realistic penalties for flying low. Uncertainty

in CV position makes Red's problem harder: if Red, at low altitudes, climbs to detect the CV too soon (as when the CV is at a greater distance than he thought) he exposes himself to the CV radar longer than necessary. In the opposite direction, if Red stays low too long, he may be nearly on top the CV or even overshoot it before climbing to attempt CV detection. He is then unnecessarily exposed to Blue's defenses, especially the PDMS. This higher exposure results from Blue's radar getting looks at the Bomber at ranges shorter than need be, and coming within range of the PDMS when it would not have been necessary with perfect position information.

q. There is a connection in the model between the decision to use CAP and the availability of Interceptors. It mirrors the real-world situation: if one uses CAP, more maintenance is needed on interceptor aircraft (on the average). Also, there is lost time going to CAP and (more importantly) returning from CAP when fuel is low. Hence there are, on the average, fewer total interceptors available for intercept when CAP is used than when CAP is not used. Red knows that this relationship exists as well as Blue does.

What is modeled is that there are two levels of numbers of available interceptors (say IA1 and IA2) for the case when CAP is used. There is also specified a probability "p" that is known also to Red. The probability represents Prob(number available = IA1), and (1-p) is Prob(number available = IA2). The value of IA2 may or may not be the same as the number of interceptors available when no CAP is chosen. Red is uncertain about the number available, but Blue knows the number available.

r. The Bombers may take off any time after the problem starts, including $t = 0$. Satellite detection, and indeed no other detection, is logically necessary for them to take off and fly out. They may rely on the Recon to detect as they fly out, or even rely on themselves detecting without the Recon's help. Whether this is reasonable depends upon the

parameters of the problem, particularly the starting value of the probability area.

s. An interceptor running out to intercept the Recon is considered no longer available for further intercepts, and makes it back to the CV safely.

t. AEW aircraft fuel is unlimited.

C. Uncertainty

The concept of uncertainty has already been introduced above, where it was emphasized that uncertainty in the ordinary sense includes uncertainty in the gaming sense and lack of knowledge about the enemy's decisions. Another way to think about it is this: in order to model (or simulate) some situation leading to a final outcome requires three kinds of variables or parameters: 1) nondecision variables known to both sides, 2) nondecision variables known only to one side, and 3) decision variables of each side. Classes (2) and (3) comprise ordinary uncertainty, and class (2) alone comprises uncertainty in this research.

An analogy that illustrates these classes is the common card game of stud poker. In this game certain cards are concealed from opponents ("hole cards") and the other cards are known to all ("up cards"). Up cards are class 1, hole cards are class 2 variables, and the poker strategies are class 3.

This section summarizes the points of uncertainty for the two sides. Note that answers to the questions are in general probabilities between zero and one, or sets of such probabilities.

1. Uncertainties for Blue

a. Has satellite detection of the CV been made by Red?

b. What is the initial probability area that Red holds on the CV?

c. What kind of ASCM will be carried by the bombers?

d. Has the Recon detected the CV yet? If so, when and how did it detect?

e. Have the Bombers taken off for the attack yet? If so, when did they take off?

f. What probability area does Red hold, at any given point during the game?

There are actually two kinds of uncertainty in this scenario and its model. The first type may be called primary uncertainty--in simulation terms, the variables which are primary uncertainty variables are selected at random at the initialization of the simulation. The other form of uncertainty is needed for convenience to decompose the game, the uncertain variables are mixtures of primary uncertainty and lack of knowledge due to decisions. A point of methodological interest is whether uncertainty can always be decomposed into the decision and nondecision components suggested above. Points a-c above are clearly uncertainty--in a simulation model these choices would be made at random upon initializing the program, independent of events in the play of the scenario itself. Point (d) already has decision considerations as well as uncertainty in it, because the answer depends upon the search mode selected by the Recon.

It may or may not be possible to disentangle the primary uncertainty from the mixture available to the players, whether this is necessary is a point of some methodological interest and concern. We will list all the primary uncertainties and enough of the "mixed" uncertainties to give the reader an idea of what is meant by the term.

Primary Uncertainties for Blue

a. What initial Recon position was selected?

b. Has satellite detection occurred already on the CV?

c. What is the initial probability area that is held by Red?

d. What kind of ASCM will be carried by the bombers?

Selected Mixed Uncertainties for Blue

Blue will effectively maintain, as a function of what he has observed and what he assumed at the outset, an estimate of some important variables of interest to him. These define the following mixed uncertainties:

a. Has the Recon detected the CV yet? If so, when and how did it detect?

b. Have the Bombers taken off for the attack yet? If so, when did they take off?

c. What probability area does Red hold, for any chosen time in the game?

2. Uncertainties for Red

Primary Uncertainties for Red

a. How many Interceptors has Blue available for intercept?

b. How much fuel does the CAP interceptor have left?

c. What were the initializing position parameters for the CAP interceptor?

Selected Mixed Uncertainties for Red

a. Has the Blue AEW aircraft detected the Bombers running out?

b. Has the CV radar detected the Bombers?

c. Is there an Interceptor now attacking the Recon? The Bombers?

D. Tradeoffs

A number of tradeoffs were discussed as the scenario was being defined in section B above. This section discusses more tradeoffs on both sides.

1. Red Tradeoffs

a. When there is no satellite detection, the attack concept is to have the Recon detect and relay position information to Red command/

control or the Bomber sufficient for the Bomber to make a successful attack. Tradeoffs involving the relative worths of the Recon versus the Bomber are inherent in the situation. The Recon would prefer to merely detect the CV and, whatever the size of the probability area determined by the detection, retreat from the scene to avoid being attacked by Blue interceptors. This involves the least risk to the Recon. However, the Bomber sees it differently--he would rather (especially if flying out at low altitude to avoid detection) have the Recon remain longer and thereby provide better information on the CV. He (the Bomber) can then execute an attack on the CV at less risk to himself, at the expense (from the system viewpoint) of more risk to the Recon.

Operations analysts have come to expect this tradeoff to be handled (if at all) by some sort of thresholding scheme: if the Recon can reduce the probability area to a certain size by a certain time, or by the time the bomber is at a certain range, then he can flee the scene. Otherwise he must stay. A rather surprising aspect of the present methodology is that such a scheme is not required. Given the utility values of the outcomes, and given the appropriate times to make the decision as to whether the Recon can flee, the optimal system decision for Red is made by the gaming algorithm. It may or may not turn out to be possible to interpret this decision as some kind of thresholding scheme. In any case the point is that many decisions one expects to be agonizing turn out to be routine in this methodology. The reason the decisions were thought to be agonizing in the first place is that values were not thought of or were not available in the right places in the overall tactical scenario. The proper place to value is at the end, as we are doing here.

b. Red Bombers trade detectability of the CV for their own detectability by selecting a point from which to climb to search altitude when at a low altitude during run-out. In general, the earlier this pop-up occurs the more exposure to the CV's radars and to Blue AEW and

interceptors. However, detection of the CV is enhanced as well. A later pop-up time may result in inadequate time to detect and get the ASCM launched, with the possibility of overshooting the CV entirely due to position error.

c. When to launch the ASCM is another tradeoff question. If launch occurs before the Bombers detect the CV the ASCM will have a generally lower acquisition probability on the CV although Bomber survival during egress will be enhanced. Waiting until detection of the CV by the Bomber enhances the ASCM's acquisition probability and may make the Bomber more vulnerable. Holding off even longer with launch (until the ASCM has locked on while still in the aircraft) removes one step of the ASCM's problem and enhances hit probability, but again at the expense of Bomber survivability.

d. The Bomber is allowed to break off the mission and return home if warranted in certain circumstances. Examples of such occasions are when the Bomber is certain or nearly certain of running out of fuel before arriving back at the airfield, and occasions where the danger is too great that the Bomber will be shot down.

e. When should the Recon transmit CV position information? This too is a tradeoff. Early transmission of poor position information gives the Bombers opportunity to begin the attack sooner. On the other hand, with early transmission the Recon is more likely to be killed or chased away before obtaining better position information. There is thus a tradeoff between quality of information and when the information is available, with Recon survival as another consideration.

f. When should Bombers take off to run out to the CV position or probability area? The scenario rules allow them to take off at any time, including the beginning of the game and even without satellite or

Recon detection. A run-out based on inadequate information runs the risk of not having CV position information refined sufficiently well for attack during run-out, implying the need for a lengthy (and risky) search or a return home empty-handed. On the other hand, if the Bombers wait too long, or demand information that is of too high quality, they may never run-out and the CV will pass through the area unattacked.

2. Blue Tradeoffs

a. Blue has to decide whether to have CAP interceptors whose advantages are (1) they may reduce the delay time in making an intercept on either the Recon or the Bombers, and (2) they may have some detection capability against the Bombers. Disadvantages are (1) they may increase the delay time, depending upon the approach the Bombers make to the CV and the CAP station selected, (2) the overall availability of interceptors is decreased due to increased consumption of fuel and increased maintenance, and (3) communications between Blue force elements required for CAP may be intercepted and exploited by Red to detect or localize the CV.

b. Active-passive tradeoffs ultimately come down to detectability of the unit itself versus detectability of its intended target. AEW, CAP interceptors, and the CV all have active-passive questions with these basic detectability issues underneath.

E. Decisions

This section specifies decisions to be made by Blue and Red more precisely than the narrative of Section B. These lists of decisions are still not complete. To make them so requires that (1) the time (or event) causing a game to be played be specified and (2) any functional dependence be shown. For an example of (1), the decision "will the CV go active, given that it is now passive?" can be asked at many points in the game, in theory it should be asked continuously. An example of (2), the functional dependence

of a decision on some other parameter or variable, is the Red decision "will the Bombers run out for the attack at the start of the problem?". The optimal answer may well depend upon the probability area that Red holds at the start of the problem, and the decision should therefore be posed differently: "will the Bombers run out for the attack at the start of the problem when the probability area is PA number j?". A third point is to be anticipated from the usual form of optimal game-theoretic decision, for the answers to the questions are in general in terms of probabilities. The Blue decision "will CAP interceptors be used?" will be represented in the gaming model by a probability, the probability that CAP interceptors will be used in given play of a game. This probability may have to be derived from other probabilities of more complicated joint events, as when Blue decisions (b) and (c) given below are combined into four combinations with three independent probabilities:

p_{11} = Probability of CAP interceptors and AEW

p_{10} = Probability of CAP interceptors and no AEW

p_{01} = Probability of no CAP interceptors and AEW

p_{00} = Probability of no CAP interceptors and no AEW

The probability of CAP is now $p_{11} + p_{10}$ and the probability of AEW is $p_{01} + p_{11}$. The decisions follow.

1. Blue Decisions

   a. x coordinate of initial CV position

   b. will CAP interceptors be used?

   c. will AEW aircraft be used?

   d. what is the number of the CAP station to be used?

   e. will the CV use its radar at the start of the game?*

*Other opportunities for going active from passive are also given at appropriate times, these require further Blue decisions.

f. will the AEW aircraft be passive, or active (i.e., use his radar) at the start of the game?*

g. will a new CAP interceptor be sent out when one returns due to low fuel state?

h. will an interceptor be launched from the deck to orbit the carrier in anticipation of the of the raiders' arrival?

i. will the Recon be intercepted by CAP interceptor following detection by Blue?

j. will the Recon be intercepted by a DLI following detection by Blue?

k. will the raid be intercepted by CAP interceptors? by DLI? Both?

l. will the attempt be made to shoot down the Bombers before ASCM launch, or wait and shoot at the ASCMs?

m. shoot-look-shoot versus salvo questions

n. will the Blue interceptors be passive as long as possible in their approach to their targets, or will they use their AI radars from a greater distance?

o. will and Interceptor, or will PDMS instead, engage an ASCM when both are capable of destroying it?

2. Red Decisions

a. will the Recon's radar be on at the start of the problem?**

b. do the Bombers begin to run out for an attack at the start of the problem?**

c. will the Recon transmit the CV position to Red command/control as soon as detection occurs?**

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* Other opportunities for going active from passive are also given at appropriate time, these require further Blue decisions.

** The same decisions are also made at other points in time during the play of the game.

d. will the bombers run out for attacking the CV as soon as position information is available from Red command/control?*

e. will the Recon break off surveillance and flee the area?*

f. attack run-out parameters (all from Bomber position at a designated time or event):

- course to fly
- cruise altitude
- distance to cruise at cruise altitude
- diversionary angle**
- point to begin climb to altitude for final search for CV
- altitude for final search
- maximum time to search for CV before breaking off search

g. ASCM launch decisions:

- from what range should the ASCM be launched?

F. Scenario Events

Major scenario events are the following:

Recon

- Recon detects CV
- Recon detects a CAP aircraft
- Recon notifies raiders of existence and position of CAP aircraft
- Recon switches from passive to active (i.e., turns radar on)
- Recon transmits CV position information to Red command/control
- Recon stops search pattern and maintains range to CV
- Recon closes CV down the bearing line to the CV

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* The same decisions are also made at other points in time during the play of the game.

** The angle bombers may turn through around the CV in order to avoid a direct approach. A diversionary angle of 180° corresponds to an approach to the CV from the direction opposite the Red airfield.

- Recon flees the area and provides no further position information
- Recon relays refined position information to raiders or Red command/control
- Recon is killed by a Blue interceptor's air-to-air missile (AAM)

<u>Bombers</u>

- Take off and begin run-out to the CV
- Drop down from cruise altitude to an altitude to avoid detection
- Begin to circle CV at range outside detection to give Blue an uncertain attack direction
- Close CV following the circling maneuver; go in for the attack on the CV
- Detect the orbiting CAP's communications with the CV
- Pop up for a final search to detect the CV
- Launch ASCMs
- Head for airfield following successful launch of ASCM
- Break off mission without launch of ASCM or go home
- Killed by AAM
- Run out of fuel

<u>ASCM</u>

- ASCM is launched
- ASCM locks onto CV
- ASCM shot down by AAM
- ASCM shot down by PDMS
- ASCM hits the CV
- ASCM misses the CV
- ASCM falls into the ocean, out of fuel

The CV and PDMS

- Turns radar on following a passive period
- Intercepts the first detection message from Recon to Red command/control
- Intercepts later CV-position messages from Recon to Red command/control
- Launches an Interceptor to replace CAP
- Launches an Interceptor to orbit the CV, anticipating arrival of the raid
- Launches an Interceptor to intercept raid
- Detects raider pop-up for final search on radar
- Detects raider or ASCM on PDMS sensor
- Launches Sea Sparrow at ASCM or bomber
- Sea Sparrow kills ASCM or bomber

Interceptors

- Launched to intercept Recon or Bomber of ASCM
- Launched to overhead CAP (i.e., to orbit the CV)
- Run out to intercept Recon
- Run out to intercept Bomber
- Turn on AI radar to detect and lock on to target
- Launch AAM at Recon
- Launch AAM at ASCM
- Go back to CV due to fuel limits

AAMs

- Launched at Recon or raider or ASCM
- Switch target from Bomber to ASCM or conversely
- Kill Recon or raider or ASCM
- Miss the target

III. A SEQUENCE OF GAMES

The Fleet Defense scenario in Section II, together with all the enumerated decisions, will result in a fairly complex game. However, simpler games can be defined within the same scenario by fixing decision variables and uncertain variables. We will define a sequence of simpler games, all consistent with the full scenario, in order to "soften up" the scenario and to introduce the concept of uncertainty.

Before proceeding further we point out that it is often useful to assume that a Monte-Carlo digital computer simulation program has been developed for this scenario. This program will have many inputs, among them the decision variables for Blue and Red. It will use a pseudo-random number generator to decide randomized decisions and to make choices involving uncertainty. One replication of the program corresponds to one play of the game, and terminates in one of the defined end-states, printing out the utility value for that end-state.

By itself, then, for given input values (including decision variables in particular), the program is an ordinary simulation program. By selecting particular decisions for the two sides, and varying them through all combinations, we can replicate the program often enough to define the payoff matrix for a zero-sum, two-person game. The simplest such game is 2 by 2, an example of which follows.

A. A Simple Active-Passive Game

Since this research is directed towards unsolved tactical problems such as active-passive decision problems, we select the two decisions for each side as "passive" and "active." These terms need not refer to just one of the passive versus active decisions in the full scenario, but may mean a complete strategy. A strategy is a rule specifying how the player

is to act for any given set of inputs encountered on any conceivable play of the game. Rules (or decisions) having nothing to do with the active versus passive issue have to be established arbitrarily for this game and such rules need not be deterministic.

So let us assume that active and passive have been defined for Blue and for Red, with meanings quite possibly different for the two players. In the simplest of cases there may be four outcomes, the product of two Blue conditions (the CV is hit or is not hit by ASCM) and two Red conditions (the bomber is lost or is not lost). A value (or utility value) matrix may be defined to show the value structure v_k for outcome numbers k (Table 1).

Table 1
Utility Values

Blue

		CV not hit	CV hit
Red	Bomber not lost	v_1	v_2
	Bomber lost	v_3	v_4

The v_k are inputs to the model and simulation program. We let p^k_{mn} be the probability of getting outcome k when Blue chooses strategy m(m = 1 or 2 for passive, active resp.) and Red chooses strategy n(n = 1 or 2 for passive, active respectively). Then the payoff (the m,n th element in the 2 by 2 payoff game matrix) is

$$a_{mn} = \sum_{k=1}^{4} p^k_{mn} \cdot v_k$$

and the game matrix is in Table 2.

Table 2
Active-Passive Game Matrix

		Blue	
		(b_1) passive	(b_2) active
Red	(r_1) passive	a_{11}	a_{12}
	(r_2) active	a_{21}	a_{22}

The b_1 and b_2 are probabilities of Blue choosing its passive or active strategy (respectively) and r_1 and r_2 are similarly defined for Red. Denoting the "game determinant" G by $G = (a_{11} + a_{22}) - (a_{21} + a_{12})$, the solution to this game is

$$V = \text{Value of the game}^{*} = (a_{11}a_{22} - a_{12}a_{21}) / G$$

$$r_1 = \text{Probability that Red uses "passive"} = (a_{22} - a_{21}) / G$$

$$b_1 = \text{Probability that Blue uses "passive"} = (a_{22} - a_{12}) / G$$

The two other probabilities are determined since probabilities sum to unity on both sides. If either of the computed probabilities lies outside the interval (0,1) the game has a saddle point in pure strategies that can be found by simple inspection.

B. An Active-Passive Game Under One-Sided Uncertainty

We can now extend this two-by-two zero sum, two person game into a game in which uncertainty is faced by Blue. As in the full scenario, we assume that the Red bombers employ one of two types of ASCM (say type A and type B), and Blue is uncertain about the type. (Blue has all the

*The Value of the game is the mean payoff for optional play. Value and MOE are essentially synonymous here.

necessary parameters about the two types, and can make predictions as needed when type is known, but he does not know the type). Blue does know, perhaps from intelligence sources, the probabilities P_A and P_B of each type.

It is not the case that Red is free to choose the missile type for a given play of the game. If Red were free to choose the type then missile type, or the probability of choosing a missile type, would be a decision variable and not an uncertain variable. What is assumed is that the missile available to and used by Red is of type A a fraction P_A of the time and type B the rest of the time.

We proceed now to the game matrix* for this more complex and interesting game. We will build up the matrix in two steps. The first step is to duplicate the original game's row and column structure so that there are two blocks in the matrix. One block corresponds to a type A ASCM, the other to a type B ASCM. The next step is to change the elements of the payoff matrix by scaling: multiply the first block through, element by element, by the quantity P_A and the second block through similarly by p_B as shown in Table 3.

*What we actually show is the main part of the linear programming table for the constrained matrix game.

Table 3
Game Matrix When Blue Is Uncertain

			Blue		
			(b_1) Passive	(b_2) Active	
Red	(ASCM Type A)	(r_1^1) Passive	$p_A\ a_{11}$	$p_A\ a_{12}$	Block 1
		(r_2^1) Active	$p_A\ a_{21}$	$p_A\ a_{22}$	
	(ASCM Type B)	(r_1^2) Passive	$p_B\ a_{11}$	$p_B\ a_{12}$	Block 2
		(r_2^2) Active	$p_B\ a_{21}$	$p_B\ a_{22}$	

An important thing to notice is that Red is solving for two cases at once while Blue is finding a generally compromised single solution. Blue is playing against an "average Red player" while Red is playing with more specific information, i.e., what the missile type actually is, not its average. Numerically, the game solution gives probabilities r_1^1 and r_2^1 for use when Red has type A and r_1^2 and r_2^2 when Red has missile type B. Blue has only one pair of probabilities b_1 and b_2. The game value in this uncertainty context is the overall average involving the p_A and p_B. Other "values" may be calculated as they are needed from the optimal probabilities and payoff elements.

Implicit in the above are the two key assumptions needed to ensure that the game value* be correct: 1) both sides know p_A and p_B (these quantities both appear in the game matrix) and 2) the actual probability of types A and B must be p_A and p_B.

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*If the players both use p_A and p_B in the game matrix, they will have strategies optimal for that pair of values, whatever the actual occurrence probabilities turn out to be. What is really important is that they both use the same values for the probabilities, so they can agree on the game matrix. Everything else in analysis follows from the matrix.

C. An Active-Passive Game with Uncertainty on Both Sides

We next extend the previous game with one-sided uncertainty to a game where there is uncertainty on both sides. Assume that Blue has three possible levels of available interceptors, say I_1, I_2, and I_3. The I_j are given integers, known to Red as well as Blue. Even leaving aside the decision as to whether or not to have CAP interceptors, the number of available interceptors will be a random variable. The probability that the number of available interceptors will equal I_1, I_2, or I_3 will be p_1, p_2, and p_3 respectively. When Blue is in the midst of the game, however, he knows how many are available, i.e., he will know which of the three cases holds. Red will know only the p_j, i.e., Red will be uncertain about the number of available interceptors.

Table 3 for the one-sided uncertainty case was not shown in full generality, and the more general form should be given before proceeding. Actually, the rows in block one are computed under the hypothesis that Red has ASCM of type A, the payoff values could have been superscripted to read a_{ij}^1 to reflect this. Similarly the elements in block 2 would have superscripts "2", and the computation done under the hypothesis of type B. This is one generalization, the superscripting of the different blocks to reflect differing Red hypothesis.

A second generalization is that the strategy labels for Red need not be the same from block one to block two, for the strategies may depend upon the missile type Red has. The number of strategies need not even be the same in the different blocks. The different blocks really represent different weighted games that Blue must find a compromise solution against.

Still more has to be said about this uncertainty concept. The format shown in Table 3 is used to solve a game in which Red hypothesizes that he has either type A or type B. That is, Red does not yet know

the type but wants to pre-plan his strategy. Table 3 is the format to use for this, where p_A is the probability that he expects to get type A. If he does get type A, he need not calculate further, he already has pre-calculated the optimal strategy for that case.

Table 3 is also the game matrix to use if Red already knows the missile type. If it is type A, Red uses the solution for type A and ignores the type B solution as being irrelevant. Why should Red be concerned with the missile type B situation at all when he knows he has type A? The answer is related to what Blue is assuming, which is that type A will be used with probability p_A and type B will be used with probability p_B. It is easy to show that Red cannot improve upon his optimal solution derived from the Table 3 game when Blue plays his optimal strategy, which is another way of saying that Red cannot improve upon his hypothesized strategies when he finally knows what he has.

Returning to two-sided uncertainty, Table 4 shows the game matrix for the case where each side is uncertain about a single quantity. There is now a double-block structure, with blocks constructed for all combinations of the uncertain variables. Payoff variables from the original game (the a_{ij}) are now scaled by multiplying by the joint probability of the uncertain variables. We assume independence of the uncertainties so that the joint probabilities are of the form $p_i \times p_A$ or $p_i \times p_B$, where i=1, 2, or 3. Generic elements are shown in Table 4 in the various blocks, for example, $p_1 . p_A . a_{ij}$ in the upper left block represents the two-by-two matrix $\{a_{ij}\}$ scaled by $p_1 \times p_A$.

In summary, Table 4 illustrates the form of the game matrix under the two-sided uncertainty when there is a single uncertain variable for each side and the game is at single stage or level. The next step in this sequence is to increase the number levels to two.

Table 4

SYMMETRIC UNCERTAINTY GAME

			BLUE	
		I_1 Passive Active	I_2 Passive Active	I_3 Passive Active
RED	Type A — Passive / Active	$\{p_1 \cdot p_A \cdot a_{ij}\}$	$\{p_2 \cdot p_A \cdot a_{ij}\}$	$\{p_3 \cdot p_A \cdot a_{ij}\}$
	Type B — Passive / Active	$\{p_1 \cdot p_B \cdot a_{ij}\}$	$\{p_2 \cdot p_B \cdot a_{ij}\}$	$\{p_3 \cdot p_B \cdot a_{ij}\}$

Next, we move to the search game at the first level. In the formulation of this game, there are quantities presumed to have been found in level 2 (end-game) and in the aggregated Recon-not-killed state as well. The optimal game values at these states will be determinants of the payoff matrix elements for the game in search. Payoff values in search also depend upon the uncertainty numbers q_i in end-game.

In theory, at least, the q_i are functions of the strategies in level 1, and we use notation such as q_i (passive, passive) when both Blue and Red are passive in level 1. Let both Blue and Red have strategies "passive" and "active" in level 1 as before, and leave off the uncertainty in level 1. Then each pair of strategies, one Blue and one Red, determine two probabilities p_{rk} and $\overline{p}_{rk}$, where p_{rk} denotes "Recon killed" and $\overline{p}_{rk}$ is $1-p_{rk}$, the probability of the complementary event. Writing $p_{rk} = p_{rk}$ (B,R) $= p_{rk}$ (Blue strategy B, Red strategy R), and v_3 for the game value at the Recon-not-killed level, the payoff b_{11} for Blue and Red both passive is calculated from $b_{11} = p_{rk}$ (passive, passive) + $\overline{p}_{rk}$ (passive, passive) x v_3 (q (passive, passive)). Other b_{ij} are similarly defined.

We begin with the end-game. Assume Blue's strategies from here on to be labeled active and passive, where the meanings may have changed

from level 1. We will leave Red's strategies in symbolic form, say S_1, S_2, and S_3. The S_i spell out rules which include, in particular, whether Red breaks off the mission and returns home without detecting the CV, presses on in his mission using the probability area as long as possible before doing his own search, or climbs to search altitude to search for the CV.

The principal Blue uncertainty at the start of end-game is probably the probability area held by Red. We approximate the uncertainty distribution by assuming three possible PAs with probabilities q_1, q_2, and q_3. The q_i are part of the iterative loop over the whole game and will accordingly vary from iteration to iteration.

If Red has no uncertainties, the game matrix is that shown in Table 5. Instead of multiplying through by the uncertainty probabilities q_i and putting the q_i in the body of the table as before, we have made marginal notes on the right to indicate scaling.

Table 5

GAME MATRIX FOR END-GAME

			Blue		
			Passive	Active	
		S_1	a_{11}	a_{12}	
	(pA_1)	S_2	a_{21}	a_{22}	← scale using q_1
		S_3	a_{31}	a_{32}	
Red	(pA_2)	⋮	⋮	⋮	← scale using q_2
		S_1	a_{11}	a_{12}	
	(pA_3)	S_2	a_{21}	a_{22}	← scale using q_3
		S_3	a_{31}	a_{32}	

IV AN EXAMPLE ILLUSTRATING THE OVERALL ALGORITHM

A. Introduction

This section presents an example in order to introduce some necessary concepts and illustrate aspects of the overall gaming algorithm, particularly the dynamic programming aspect. The example is a very greatly simplified case derived from the Fleet Defense scenario in Section II. In Section V , the methodology and model will be generalized and formalized. The formalism is then employed to develop models for the full Fleet Defense scenario in Section VI . The reader interested in approach in its greatest generality will find it in Section V .

The example begins with a *state diagram* in the strict Markovian sense of state. Transitions are made at time t=1, 2, and 3 based on strategies representing decisions at times 0, 1, and 2. Diagrams in later sections will not have state as their basic elements (ellipses or circles). The usual practical trouble with state is that its description is too complex to be shown on a diagram. This example is therefore quite special.

The overall approach is iterative--each iteration results in a "solution" composed of strategies for each player at each state. (The initial iteration is made using strategies chosen arbitrarily.) Having finished a backwards iteration using dynamic programming, one uses the solution to determine new transition probabilities that will be needed in the next backwards iteration. A backwards iteration starts at the largest unevaluated time (t=2 in the example) and solves the state games for t=2, then moves to t=1 and solves its state games, and finally

moves to t=0 to solve that state game. The new solution has been found and can be compared with the old for a convergence test. Iteration continues until the old and new solutions are sufficiently close to each other, i.e., until the convergence test is passed.

B. An Example Illustrating the Dynamic Programming Algorithm

Consider a Markov model which starts at time zero in the search state, with Red and Blue searching for each other. Transitions are made at time 0, 1, and 2 to other states. Symbols are defined as follows:

B	means	Blue detects Red (and Red does not know it)
R	means	Red detects Blue (and Blue does not know it)
(B,R)	means	Blue detects Red and Red detects Blue (but neither knows of the other's detection)
.	means	Non-B or non-R, depending upon the context. Also translates as "nothing" or "empty".
4	means	A terminal state (Blue kills Red)
5	means	A terminal state (Red kills Blue)

When symbols are strung together, the successive symbols are time-related. For example, BR means that Blue detected R at time t=1 and then Red detected Blue at time t=2. Figure 2 shows the state tree for the example, by time t=3 some terminal state has always been reached. Table 6 is easily derived from the figure, it shows the states that are possible for each time from t=0 to t=3.

Figure 2

State Diagram

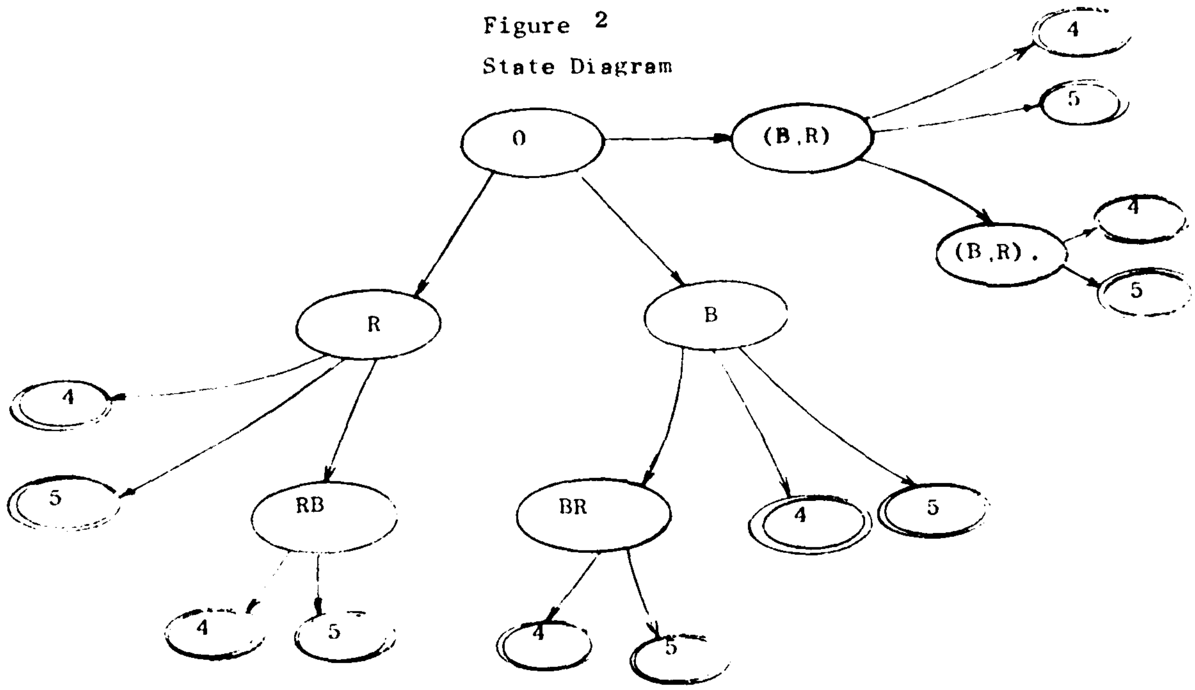


Table 6

Reachable states at times t=0, 1, . . 3

t	game states	terminal states
0	0	
1	R, B, (B,R)	
2	RB,BR,(BR).	4, 5
3		4, 5

At each time t, Blue and Red each have a certain amount of information about the game and how it has progressed, these are called their information sets.* For example, at time t=1 in state B, the information set for Blue is B (Blue has detected Red) and the information set for Red is ".", meaning that nothing has happened from Red's point of view. In state BR at time t=2, the information set of Blue is "B.", meaning Blue detect Red at time t=1 and nothing happened at time t=2, while Red's information set is ".R"; nothing happened at t=1 but Red detected Blue at t=2.

We will now tabulate the possible information sets in matrix form so that states may be related to information sets. Table 7 shows information sets on the row and column margins for times t=1 and t=2. An "X" indicates an impossible situation in the body of a matrix. Having the information sets shown on the margins, the states may now be associated with pairs of information sets. Each state can be compatible with only one pair of information sets. In this example, it just happens that there is no more than one state shown for a given information set pair at a time t, this will not generally be true.

Table 7
Information Sets And Compatible States

RED \ BLUE, t=1	.	B
.	X	B
R	R	(B,R)

RED \ BLUE, t=2	B.	.B
R.	(B,R).	RB
.R	BR	X

*These and other new terms are defined carefully in the next section.

Now gaming considerations can be introduced. Because a state is equivalent to a problem definition, payoff matrixes are defined at states. Strategies may sometimes be chosen with full knowledge of the state (i.e., with full knowledge of the payoff matrix) but more commonly they must be selected without full knowledge of the payoff matrix. At State 0 (Search) there is no problem with information or uncertainty, and an ordinary matrix game will suffice. Strategies in this game result in transition probabilities which govern the transitions to states at time t=1. At time t=1, the players must play based on their knowledge, that is on their information sets. If Blue has detec d at t=1, then he can use certain strategies, but those strategies must be the same for all states giving him the same information set. That is, consulting the right column in the top of Table 7, Blue's strategies in information set B must be used in states B and (B,R). Blue must choose a strategy without knowing which state the system is actually in. Red is in a similar position if in information set R, since the state may be either R or (B,R). The two players are better off in terms of knowing the state when they have not detected, specifically if one has not detected he knows the other has detected. For time t=2, the situation is similar, some of the games are played without knowing state while others are played knowing the state.

At each state, then, is a payoff matrix. Let us denote the payoff matrix at state s by A^s with elements a^s_{mn}. The correct formulation of the games at all states for a given time t turns out to involve weighting the payoff matrixes at the states with the probabilities of be being in the states at t. For example, choose t=1 and consider the top table of Table 7. The possible states at t=1 are B,R, and (B,R) and the probabilities of occupying these three states at t=1 therefore sum to one. The constrained game matrix for this situation is shown in Table 8, where strategy probabilities for each side are shown

symbolically on the margins. At t=1, Red has three strategies if his information set is empty (".") and two strategies if it is R. Similarly, Blue employs three strategies for "." and four strategies for B. The X and Y's denote probabilities for Blue and Red (respectively) in general, the superscripts are information sets, and the subscripts the strategy numbers. The bottom subtable shows the CMG matrix for all states at t=2 in a similar manner. In the linear programming tableau to solve the games, the constraints must of course be added to force probabilities for each information set for each side to sum to unity.

Table 8
CMG Matrices For Times t=1, 2

		$X^{.}_1$ $X^{.}_2$ $X^{.}_3$	X^B_1 X^B_2 X^B_3 X^B_4
t=1	$Y^{.}_1$ $Y^{.}_2$ $Y^{.}_3$	○	Pr(state=B at t=1)·A^B
	Y^R_1 Y^R_2	Pr(state=R at t=1)·A^R	Pr(state=(B,R) at t=1)·$A^{(B,R)}$

		$X^{B.}_1$ $X^{B.}_2$ $X^{B.}_3$	$X^{.B}_1$ $X^{.B}_2$
t=2	$Y^{R.}_1$ $Y^{R.}_2$	Pr(state=(B,R). at t=2) x $A^{(B,R).}$	Pr(state=RB at t=2) x A^{RB}
	$Y^{.R}_1$ $Y^{.R}_2$ $Y^{.R}_3$	Pr(state=BR at t=2) x A^{BR}	○

In Table 8, the probabilities of state occupancy are known from the previous iteration. We have to explain the derivation of the other elements in the tables, the elements of the payoff matrixes A^s at states s and times t=1,2. The elements of A^s are derived from transition probabilities out of s and state-game values at states that can be reached from s.

In Table 9, the block structure of Table 8 is repeated. Since transitions at t=2 can occur only to states 4 and 5, elements in this table should be pairs of transition probabilities: the probability of transitioning to state 4 and the probability of transitioning to state 5. Since the probabilities of transitioning to states 4 and 5 sum to unity, however, only a single probability needs to be displayed.

The general notation for the probability of transitioning from state s at time t to state d at time t+1 when Blue makes strategy choice n and Red makes strategy choice m is $p_{m,n}^{s,d}$.

The two blocks on the top of the table are filled out in full with their respective dimensions. A third block on the bottom is explained in words. The last block on the bottom is zero because there was no state compatible with the player's information sets defining this block.

The final linkage to describe is between the transition probabilities and the elements $a_{mn}^{s,d}$ of the payoff matrix at state s. The payoff element is taken to be the average value of a game starting at s, defined to be the average of the game values at states d reachable from s at time t+1. Weights used in the averaging are the transition probabilities. Symbolically,

$$a_{m,n}^{s,d} = \sum_{d \in D_s} v^d \times p_{m,n}^{s,d}$$

where D_s is the set of states reachable at t+1 from state s at t .

Table 9
Transition Probabilities at t=2

	$X_1^{B.}$	$X_2^{B.}$	$X_3^{B.}$	$X_1^{.B}$	$X_2^{.B}$
$Y_1^{R.}$	$p_{1,2}^{s,4}$	$p_{1,2}^{s,4}$	$p_{1,3}^{s,4}$	$p_{1,1}^{s',4}$	$p_{1,2}^{s',4}$
$Y_2^{R.}$	$p_{2,1}^{s,4}$	$p_{2,2}^{s,4}$	$p_{2,3}^{s,4}$	$p_{1,1}^{s',4}$	$p_{1,2}^{s',4}$
$Y_1^{.R}$	nine elements $\{p_{m,n}^{s'',4}\}$				
$Y_2^{.R}$					
$Y_3^{.R}$					

s=(B,R). on left
s'= RB on right
s''=BR below on left

Given models for obtaining the transition probabilities, the payoff matrix in the bottom of Table 8 can be filled out to start a linear programming tableau. Constraints must be added to make the X's and Y's in each block sum indepently to unity. This game can then be solved. Having solved the game at t=2 using the a above, game values can be associated with the states. At state BR for example, one takes the ij double sum of the products of strategy probabilities and payoff elements from A^{BR}; the result is the state-game value v^{BR} for state BR. This value is saved, replacing the previous stored state-game value for state BR. Optimal strategies for the information sets are also stored for each player at time t=2.

$$\sum_{n=1}^{3} \sum_{m=1}^{3} X_n^{B.} Y_m^{.R} a_{mn}^{BR} = V^{BR}$$

State-game values are similarly defined for the two other states that can be occupied at t=2. The algorithm now moves to t=1 and employs the game-state values at t=2 that were just computed. Information-set strategies and game-state values are determined for t=1 by again using a linear

program to solve a CMG. The search state (0) can be then be solved, ending the iteration.

As the next two sections show more precisely, this example generalizes nicely to other loopless Markov processes. At any given time, there is a set of reachable states, and from these information sets can be defined for Blue and Red. A matrix showing these information sets, and states compatible with them, can be derived. From an earlier iteration, the probabilities of occupying the reachable states may be found and used to weight the payoff matrixes at the states. In a given block (defined by a Red information set and a Blue information set) the payoff matrix is the sum of products over all states compatible with the information sets, the product being Prob (occupying the state) x (payoff element for the state). A linear program solves this constrained matrix game for all information sets at time t or equivalently for all states reachable at time t. The state-game value at a state is the double sum of information set-strategy probabilities times payoff elements for the state, and these values are used for the solution at time t-1. An iteration ends when the t=0 solution is obtained. A comparison of some or all of the quantities which vary from iteration to iteration is made to determine whether convergence has been achieved. Quantities that vary from iteration to iteration are: information set-strategies for both sides at each information set at each time, game-state values at each state at each time, and the overall game value.

V DEFINITIONS AND NOTATION FOR THE GAMING ALGORITHM

This section generalizes and formalizes the model illustrated by example in the previous section. Algebraic notation is introduced as appropriate. Since an example has already been given, the explanatory information is intentionally cryptic. Some special remarks apply concerning timing of transitions and of decisions. One may think of the process as being sampled at integer times t (t=0,1,2,...). At time t the process is in some state, and based on state, a decision is made. This decision influences transitions made from the time of the decision until the time of the next sampling. For definiteness, one can think of a decision at t as being made just <u>after</u> time t and the transition influenced by the decision as occurring just <u>before</u> time t+1,

Thus, decisions are made at t=0,1,2,... and transitions are made at t=1,2,3,...

1. Events are occurrences observable by one or both players. Actually, events are of two types that can be classified as decision and nondecision. A decision event may be "Blue decides to intercept the Recon" while a nondecision event is such an occurrence as "Blue detects Recon." When "event" is used without qualification, it usually means a mixture of the two types.

2. An ordered sequence of events from the starting condition of the problem is called a <u>path</u> or an <u>event-state</u>.

3. Certain nondecision events for the scenario can be used as basic elements in an <u>event-flow diagram</u> for the scenario. This is

accomplished by connecting events together with arrows in the ways meaningful in the problem context. The events are usually shown by ellipses and the name of the event (or its code) placed inside the ellipse. A path (in the sense of (2) above) is a path in this diagram from the ellipse representing the starting state to the selected event on an event-flow diagram.

4. An <u>event-state tree</u> EST* is defined from the event-flow diagram EFD by path tracing, using the rule that no ellipse (event) in the EFD should be entered more than once. This requires duplication (i.e., duplicate ellipses with the same label) on the EST. The diagram that results is technically called a tree because there is a unique path between any two ellipses. In particular, there is a unique path from the starting ellipse to any selected ellipse, hence the selected ellipse can be identified with the entire path. One can think of the successive events along a path as being accumulated and placed in sequence from left to right in order of occurrence. Thus, the label of an ellipse at an event may be 1AmDe, which means: the history of this path is as follows: the starting event was 1, then event A happened, then event m happened, and then event D. Event e just happened, following D.

5. The idea of state is as important as is formal definition. In a practical sense, <u>state</u> is information that is needed to define a problem. There is no requirement that all elements in the definition be deterministic. In this report, a state is considered to have three components that together define a problem. Indeed, in the present context this is a meaningful tactical problem and not mathematical abstraction.

The three components of state are:

a. a path h (i.e., a path in the EST)
b. selected decision events (BDE and RDE)
c. selected random variables (or probability distributions (RV))

---

* later this is called an <u>aggregated</u> Event State Tree because the one defined here will be refined

Symbolically,

state = s = (a path) + (selected decision events) + (selected random variables)

or

s = h + BDE(h) + RDE(h) + RV(h), where "+" means union

- The BDE and RDE are analyst-determined for the path as are the RV(h).
- State s, together with the scenario model, determine initial conditions for a tactical problem. The principal information is given by the events in the path h and is augmented by selected decision events for Red and Blue in combination.
- The selected decision events determine the multiplicity m(h) of event-state h. Still more information (RV(h)) may be given statistically in terms of means, means and variances, or probability distributions. Geometric information is generally transferred in this way. Processing at path h, i.e., at the states with this h, effectively integrates out the RV(h).

6. The information set for Blue at state s is denoted by I or I(s), for Red it is J or J(s). I(s) and J(s) are uniquely determined by s as implied by the notation.

At any time t the set of reachable states is denoted by S(t). The collection of Blue information sets is exhaustive and mutually exclusive, i.e., each s in S(t) is in some I and no states s is in two or more Blue information sets. Similar statements hold for the J(s) for Red.

7. When a complete set of Red and Blue strategies for all information sets at all times are defined, a simulation model for the scenario is completely defined. One can then speak of the probability of occupying a state s at time t. In particular,

$$P_t(s) = \text{Prob (state = s at time t)}$$

In the sample problem, the time t is implicit in event state h, and hence implicit in s. Notation can therefore be simplified to P(s) by dropping the t.

8. The information set matrix F(t) at time t is a matrix whose rows are the I=I(s) and columns are J=J(s) for s in S(t). Each state s can be uniquely placed into the F(t) matrix since the row is I(s) and the column is J(s). State s is then said to be compatible with I,J. Blue nor Red in general know the state s; Blue knows I and Red know J as determined by s. The states compatible with I are those in the I th row of the information set matrix F(t)--Red knows the state is one of this collection when he holds I. Similarly Blue knows the state is one of the collection in the J th column. The intersection of this row and this column determines the states compatible with both Blue's J and Red's I. Matrix elements are zero or one or more of the states s in S(t). Each s is compatible with a unique I, J combination, but a given I,J combination may have several states associated with it. Some I,J combinations may be empty, i.e., no state will be compatible with I,J. An example for I=1,2 and J=1,2,3 follows; the set S(t) consists of four states, s_1, s_2, s_3, and s_4. The matrix dimensions are 2 by 3.

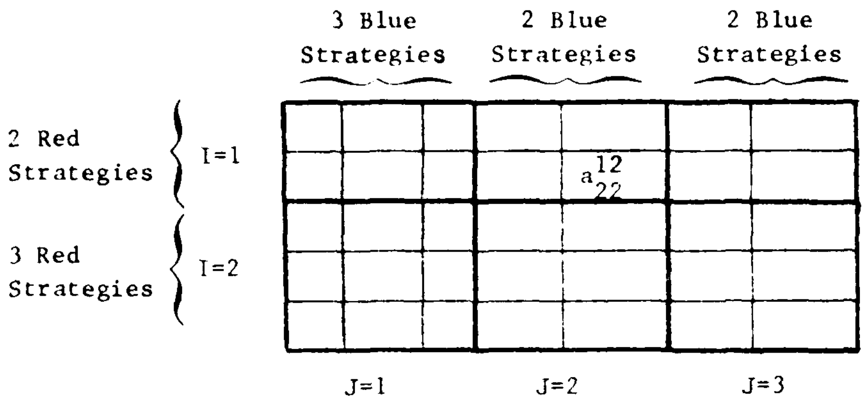
BLUE

RED		J=1	J=2	J=3
R	I=1	none	s_1	none
E D	I=2	s_2, s_3	none	s_4

The information set matrix F(t)

9. The payoff matrix at time t is denoted by M(t). It is built of blocks of payoff matrixes, where the block structure is the same in the information set matrix F(t). A given block is in M(t) is dimensioned from the strategies for Red and Blue in their information sets I and J. If Red has r strategies for information set I and Blue has c strategies

for information set J, then the I,J block in the payoff matrix M(t) is dimensioned r rows and c columns. The example information set matrix above serves as a basis showing the block and row/column structure of the payoff matrix M(t). The dimensions of M(t) are 5 x 7.



10. Elements in the payoff matrix M(t) are denoted by a^{IJ}_{mn}, the I,J referring to block and the m,n referring to the (local) indices within the I,J block. A single element a^{12}_{22} is shown in the sample above, all other positions will have a value as well. The full matrix is shown symbolically below for the example above. Zeros appear in blocks which were empty in the corresponding I,J positions in the information set matrix F(t).

$$
\begin{array}{c}
 \\ \text{I=1} \\ \\ \text{I=2} \\ \\
\end{array}
\left.
\begin{array}{ccc|cc|cc}
\multicolumn{3}{c|}{\text{J=1}} & \multicolumn{2}{c|}{\text{J=2}} & \multicolumn{2}{c}{\text{J=3}} \\
\hline
 & 0 & & a^{12}_{11} & a^{12}_{12} & & 0 \\
 & & & a^{12}_{21} & a^{12}_{22} & & \\
\hline
a^{21}_{11} & a^{21}_{12} & a^{21}_{13} & & & a^{23}_{11} & a^{23}_{12} \\
a^{21}_{21} & a^{21}_{22} & a^{21}_{23} & & 0 & a^{23}_{21} & a^{23}_{22} \\
a^{21}_{31} & a^{21}_{32} & a^{21}_{33} & & & a^{23}_{31} & a^{23}_{32}
\end{array}
\right\} \text{The matrix } M(t)
$$

11. The payoff elements a^{IJ}_{mn} in M(t) are weighted combinations of elements from other payoff matrixes. Since state s defines the initial conditions for a tactical problem, it has a payoff matrix associated with it. Elements of this payoff matrix are denoted by a^{s}_{mn}, where m is the number of the Red strategy in information set I(s) and n is the number of the Blue strategy in information set J(s). In matrix form, we write A^{s} for the a^{s}_{mn} matrix. The a^{IJ}_{mn} are formed by

$$a^{IJ}_{mn} = \sum_{s \text{ in } (I,J)} P_t(s)\, a^{s}_{mn} \qquad \text{all } m,n,\ I,J$$

where the sum is over all states s in the I,J block in the information set matrix.

It is best to think about filling up the payoff matrix in a block-by-block manner. To use the formula above, first fix I,J (the block) and vary m and n. Then vary the I,J over all blocks. From the estimated optimal strategies from the previous iteration, the probabilities $P_t(s)$ are determined and are assumed available for the a^{IJ}_{mn} formula.

12. It remains to show how to determine the payoff elements a^{s}_{mn} in the payoff matrix M(t). We assume an iterative dynamic programming algorithm working backwards in time. For a given state s (at time t) the set of possible successor states at time t+1 is denoted by D_s. At each state d in D_s, the <u>state-game value</u> v^d has already been determined, since we are working backwards in time. Let the probability of transition from state s (at time t) to state d (at time t+1) be $p^{s,d}_{m,n}$. Then the a^{s}_{mn} are found by averaging the state-game values v^d using transition probabilities for weights:

$$a^{s}_{m,n} = \sum_{d \text{ in } D_s} p^{s,d}_{m,n} \cdot v^{d} \qquad \text{for all } s,m, \text{ and } n$$

13. All that remains are the most basic sets of values in the model, the transition probabilities. These may be determined in any one of several ways depending upon the state. For clarity, we will assume they are to be found by Monte Carlo simulation from the computer program based on the flow diagrams in Appendix A. We now give a verbal description of the process used to find $p_{m,n}^{s,d}$. This process is potentially programmable, given a flexibly designed Monte Carlo simulation model.

Here is the process to find $p_{s,d}^{m,n}$. Recall that s = h + BDE(h) + RDE(h) + RV(h), which means:

> state = (path to an event) and (selected combinations of Blue and Red decisions made prior to time t) and (a joint probability distribution of either selected variables).

The path information and decision information determines certain status information in the computer program to initialize it. This means that the events and decisions that matter for the transitions out of s help initialize the Monte Carlo model.

Now the strategies corresponding to m and n (in the I,J block determined by s) are input to the Monte Carlo program to tell the players how to make decisions in the next time-step. (In general, these strategies consist of deterministic rules, but they could have random components.) We will run the Monte Carlo program only from time t to time t+1, since all values are known at time t+1 by assumption. Each replication will end in one of the successor states D, and there will be N replications. Of these N_{mn}^{d} terminate in state d, and the sum over the N_{mn}^{d} equals N. Naturally, the estimate of the transition probability p_{mn}^{sd} will be d ratio:

$$p_{mn}^{sd} = N_{mn}^{d} / N \qquad \text{for all } s,d,m, \text{ and } n$$

What do we randomize over from replication to replication? In programming terms, for what do we draw random numbers? Randomness enters in several ways:

(1) Past randomness: The random variables RV(h) convey to the model, in an aggregated manner, all randomness in the problem prior to time t. This randomness is present even for the fixed strategies in earlier steps from which the selected Blue and Red decision events are defined. Search models in particular have random outputs.

(2) Randomness over the next time step: For any given set of inputs initializing the tactical problem, further randomness is, in general, introduced by the models governing the players from time t to time t+1. If state s includes the Recon detecting the CV but not the CV detecting the Recon, for example, the CV search model will probably have a random output for any given tactical problem, and this randomness will influence the successor state. That is, the CV search model will add more randomness.

(3) Randomness in one or both of the strategies associated with m and n. It is unlikely that this kind of strategy will be used.

VI AN INTERMEDIATE FLEET DEFENSE PROBLEM

In this final section, we will use the terminology and notation of Section V to define and model a Fleet Defense problem lying somewhere between the example of Section IV and the full-blown scenario of Section II. As most as possible will be accomplished by means of diagrams to define and model this intermediate problem. Speaking loosely, this intermediate problem results from the full problem by:

(1) Eliminating CAP and AEW

(2) Having a binary probability area implicitly defined

(3) Assuming the Recon transmits detection information immediately to Red command/control instead of possibly waiting

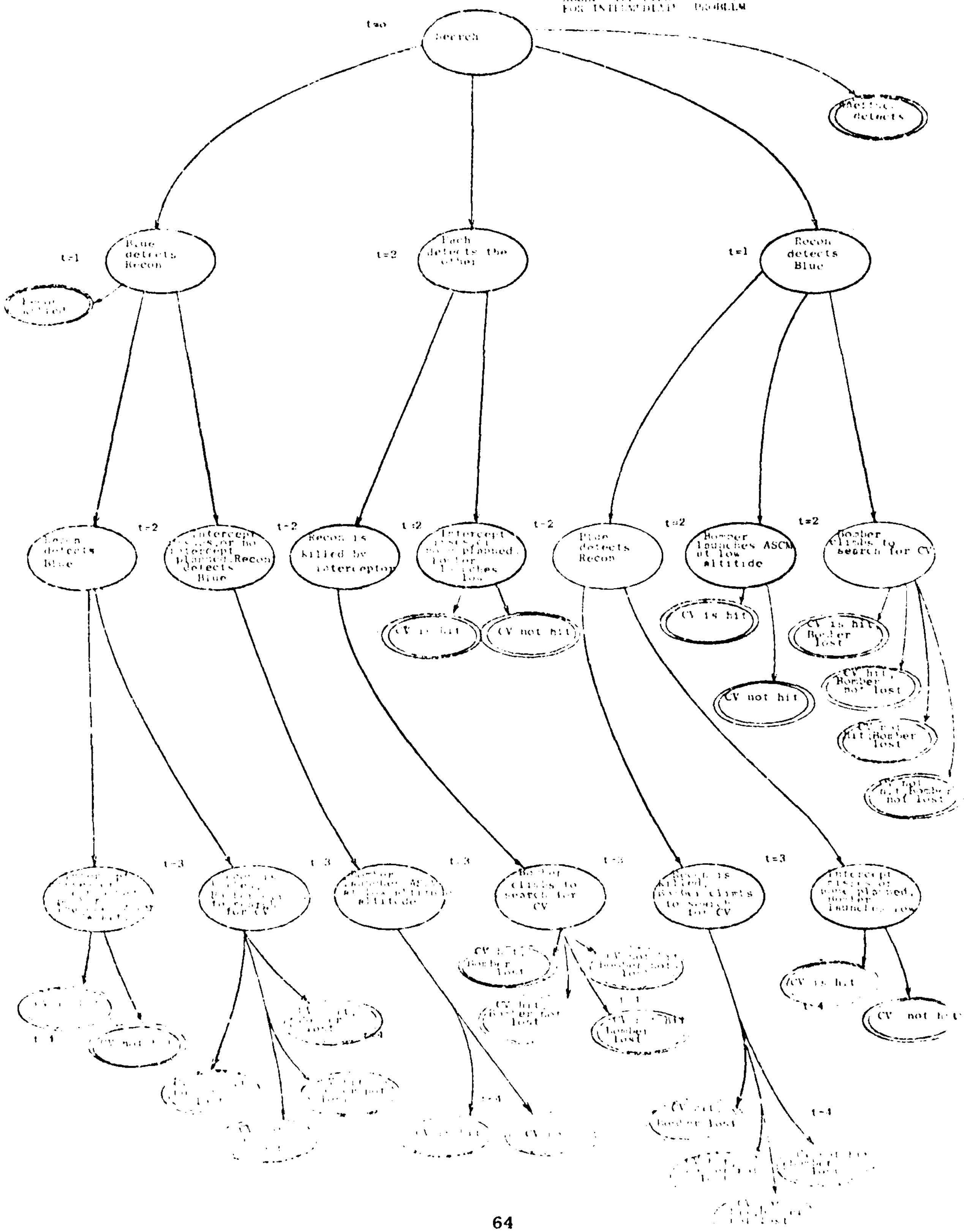
(4) Eliminating the PDMS so that interceptors are the only defense for Blue

(5) The number of Red approaches to the CV is limited to three

A series of diagrams will be presented, each with more information than the last. We will dispense with the event-flow diagram and start with the aggregated event-state diagram (AEST). This diagram will have its events at a given time shown on a single line. Word definitions of events and codes for the events will be shown on this first diagram, using the following convention: 1,2,3,... represents events fully known to both sides, uppercase letters A,B,...L represent events known only to Blue, and lowercase letters m,n,p,q...z represent events known only to Red. This convention is convenient for figuring out relationships between event-states and information sets. Figure 3 shows the AEST for the intermediate problem.

FIGURE 3

AGGREGATED EVENT STATE TREE
FOR INTERDEPENDENT PROBLEM



The next diagram* will have added information concerning strategies (decisions). These will be shown with tic-marks by the ellipses. The same decisions recur at different times and in different event-states; specifically Blue repeatedly decides whether the CV should go active (if it is still passive), and Red similarly makes active/passive binary decisions for the Red Recon.

The third diagram* builds on the second by showing the information from an element (ellipse) to elements at the next time. There are two kinds of notation, one for each of the latter two components of state. Tic marks show BDE (Blue decision event) and RDE (Red decision event) information, while a dot shows random variable (RV) information. The BDE/RDE data determine the multiplicity of the ellipse to which they go, and the multiplicity is also shown. The third diagram of this series is a fully annotated Aggregated Event-State Diagram. Figure 4 shows the annotated AEST.

By disaggregation of an aggregated EST, we mean that ellipses with multiplicity higher than unity should be replaced by the full number of ellipses, and relabeled according to the decision information. The resultant ellipses represent more refined state information and thus come closer to giving enough information to define initial conditions for a tactical problem. The disaggregated diagram is a detailed state diagram , and its primary elements are states. Beyond this, the only essential information are the transition probabilities from states at time t to states at time t+1. Figure 5 is a state diagram for the intermediate problem.

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* Note in proof: the second diagram is not shown in the text, we have gone directly to the third (as Figure 4)

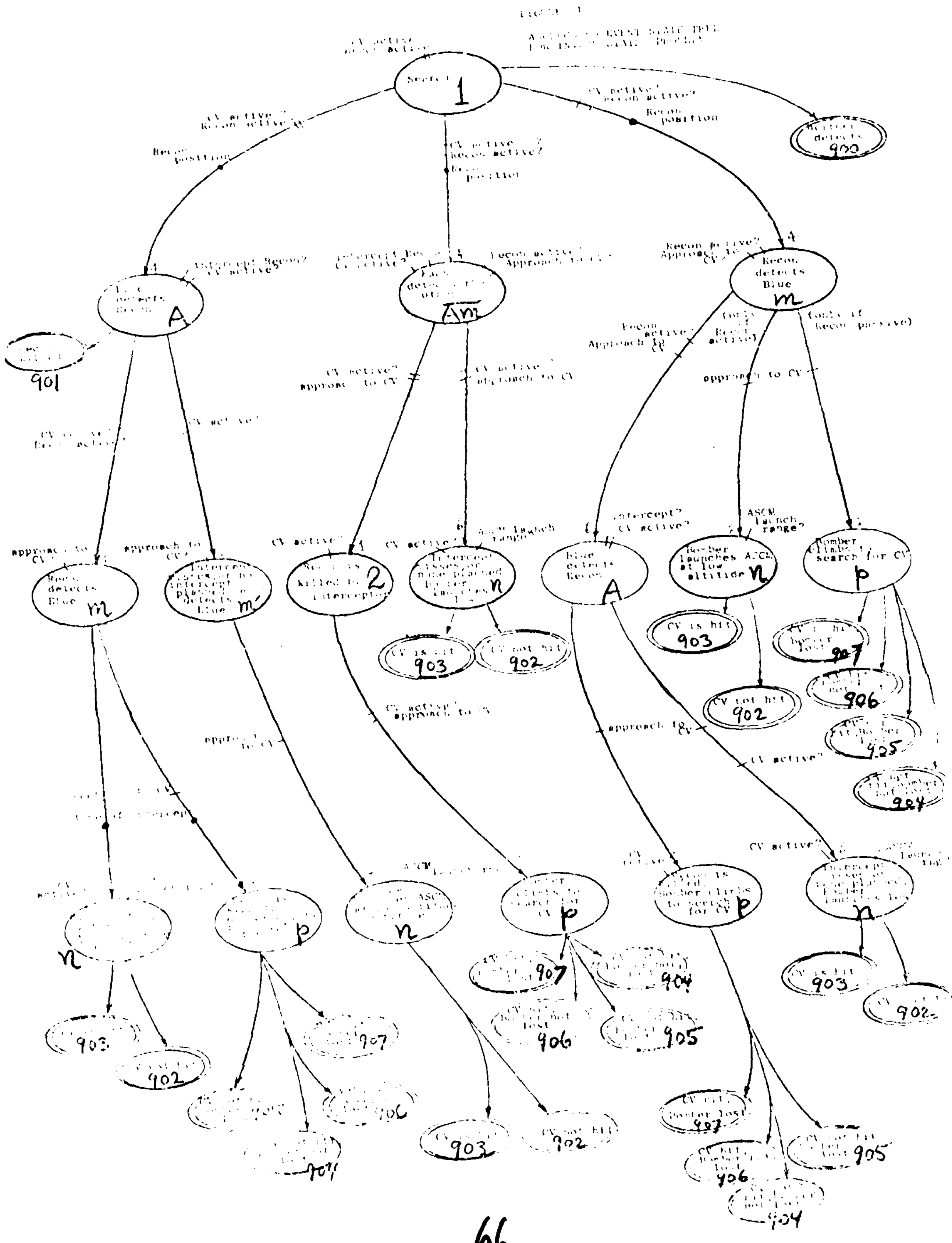
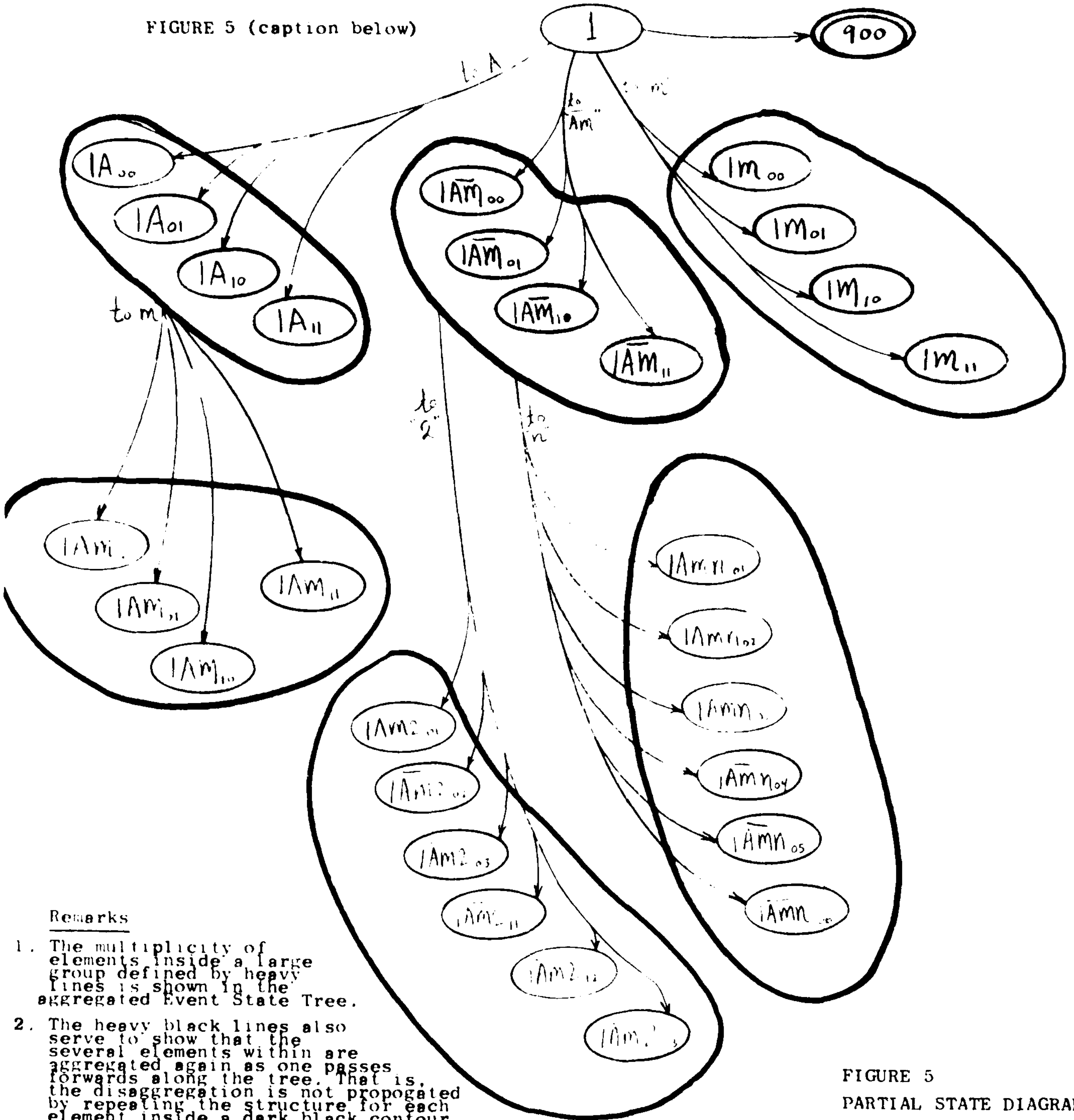


FIGURE 5 (caption below)



Remarks

1. The multiplicity of elements inside a large group defined by heavy lines is shown in the aggregated Event State Tree.
2. The heavy black lines also serve to show that the several elements within are aggregated again as one passes forwards along the tree. That is, the disaggregation is not propogated by repeating the structure for each element inside a dark black contour.

FIGURE 5
PARTIAL STATE DIAGRAM
FOR INTERMEDIATE
PROBLEM

Appendix A

FLOW DIAGRAMS FOR A MONTE CARLO FLEET DEFENSE MODEL

FIGURE A-1

POINT DEFENSE MISSILE SYSTEM (PDMS)
AND SEA SPARROW MODEL

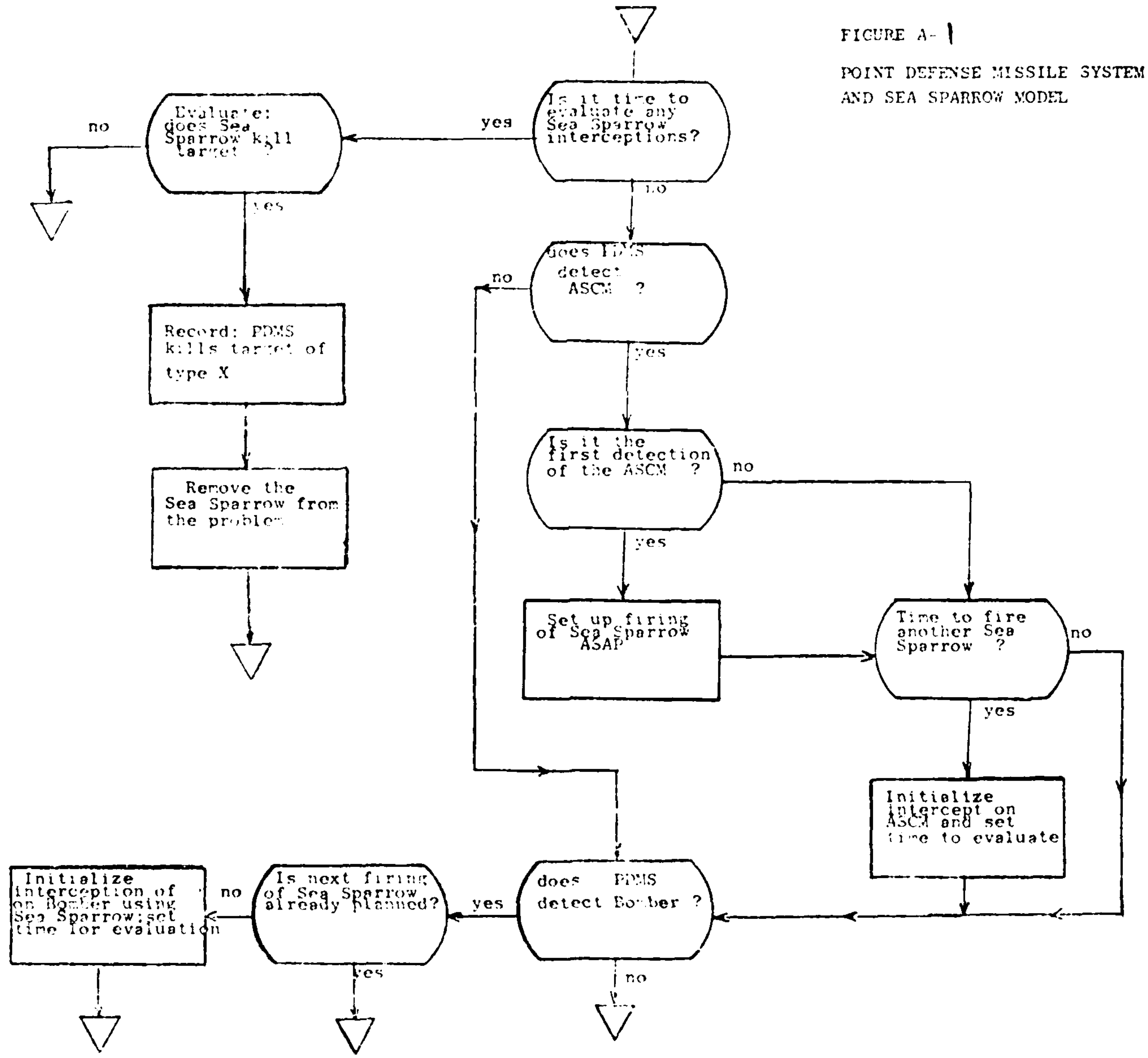


FIGURE A-2

FLOW DIAGRAM FOR THE RECONNAISSANCE AIRCRAFT

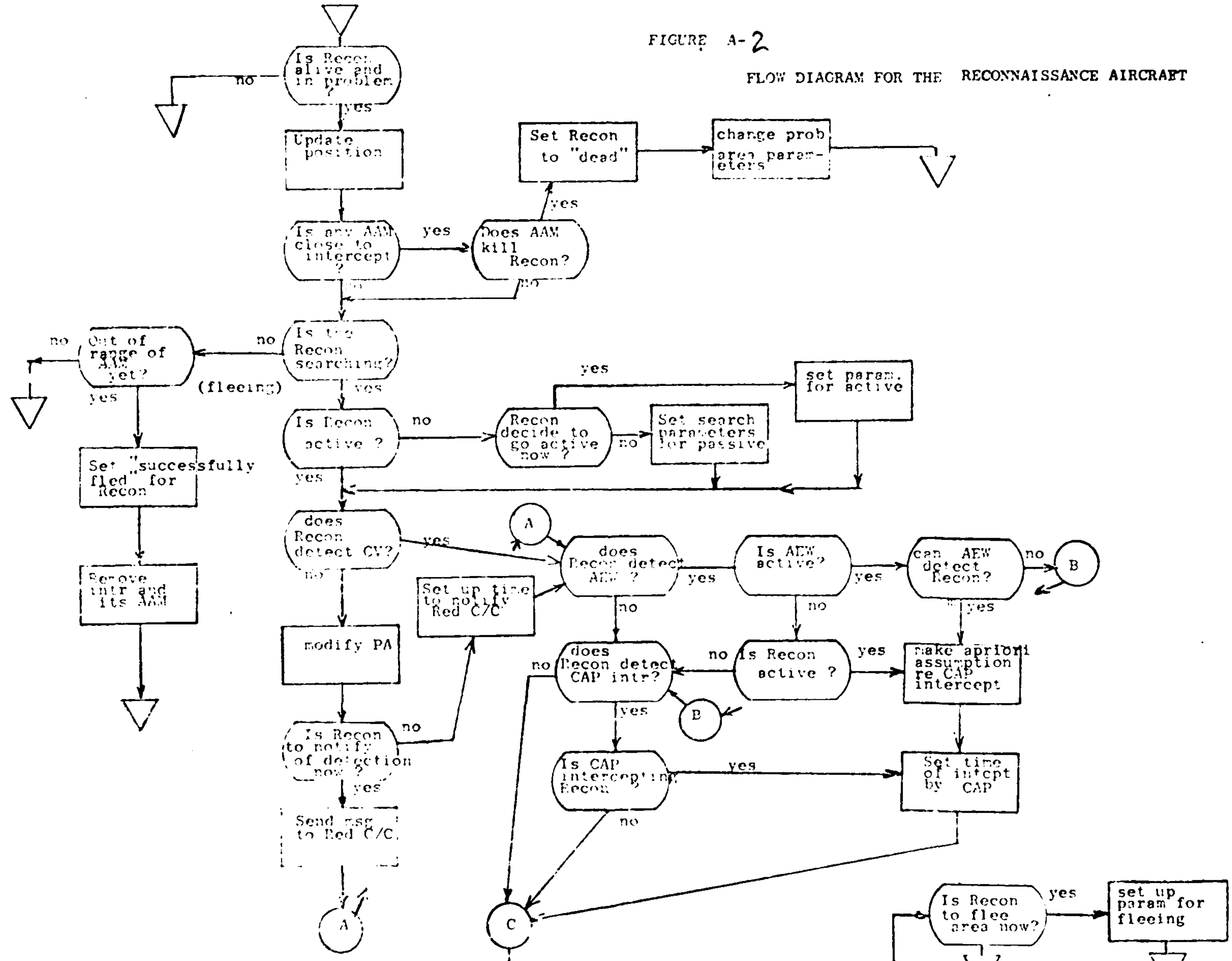


FIGURE A-3
FLOW DIAGRAM FOR THE
CV MODEL

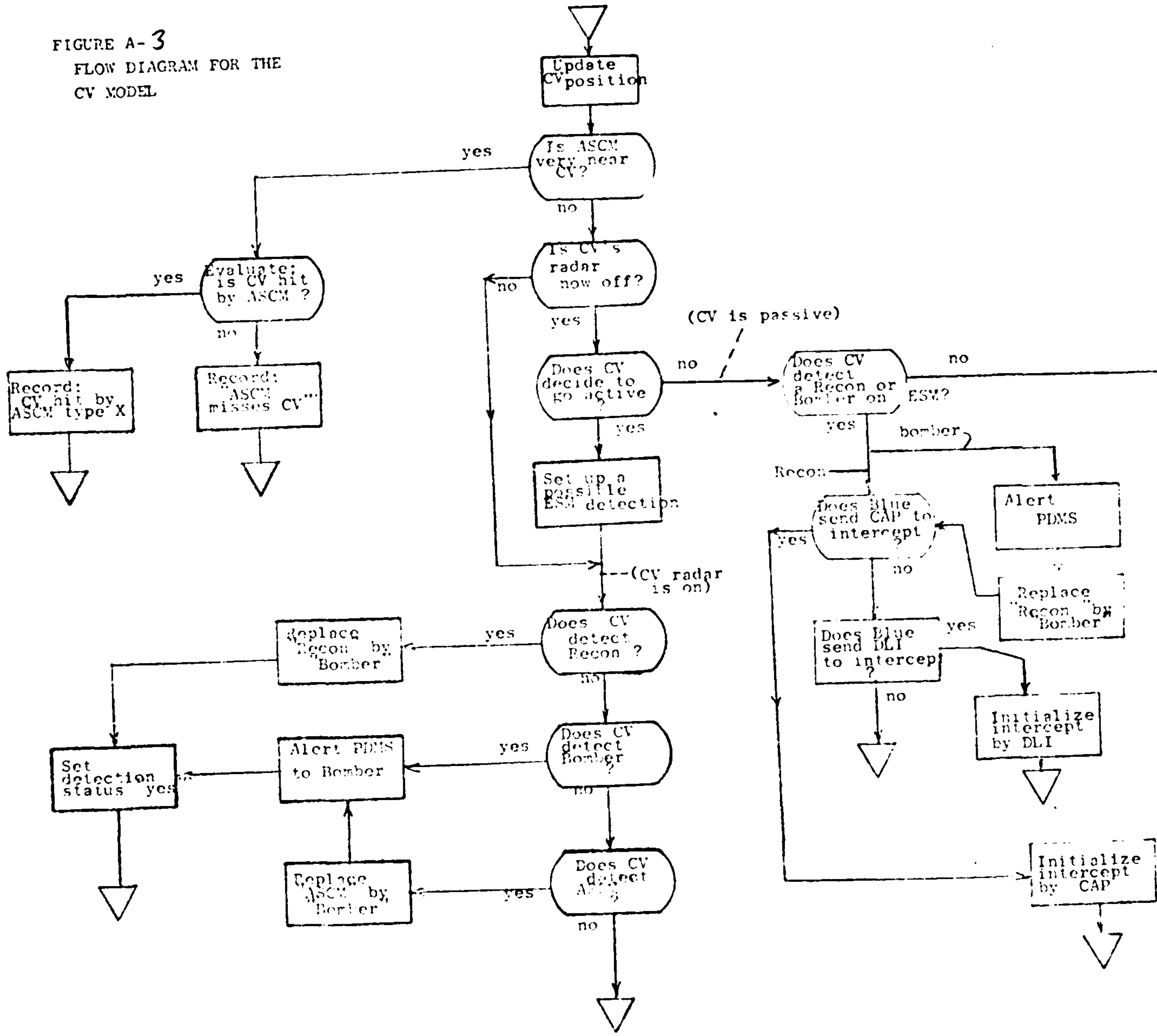


FIGURE A-4

FLOW DIAGRAM FOR BOMBER MODEL

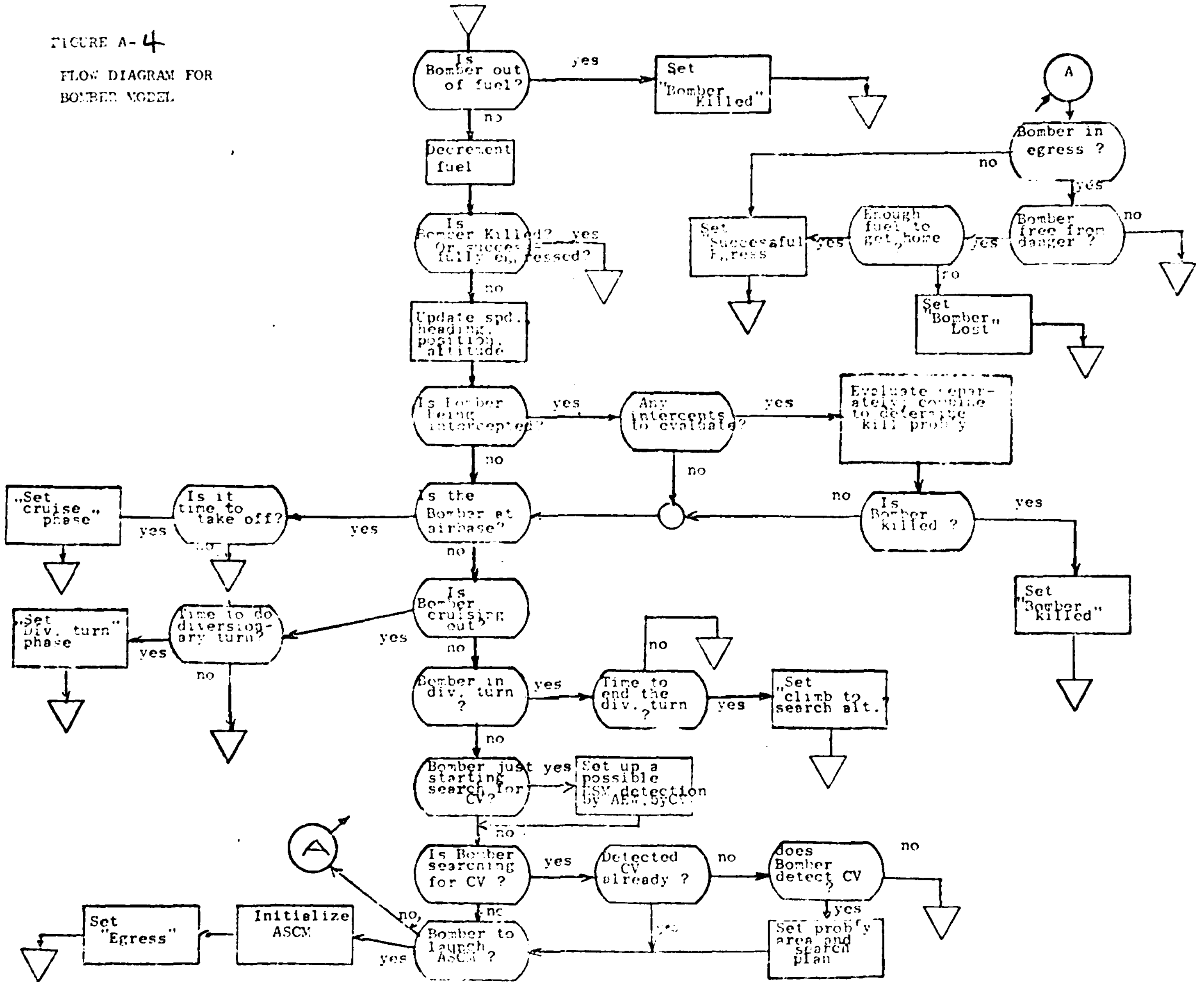


FIGURE A-5

FLOW DIAGRAM FOR THE ASCM MODEL

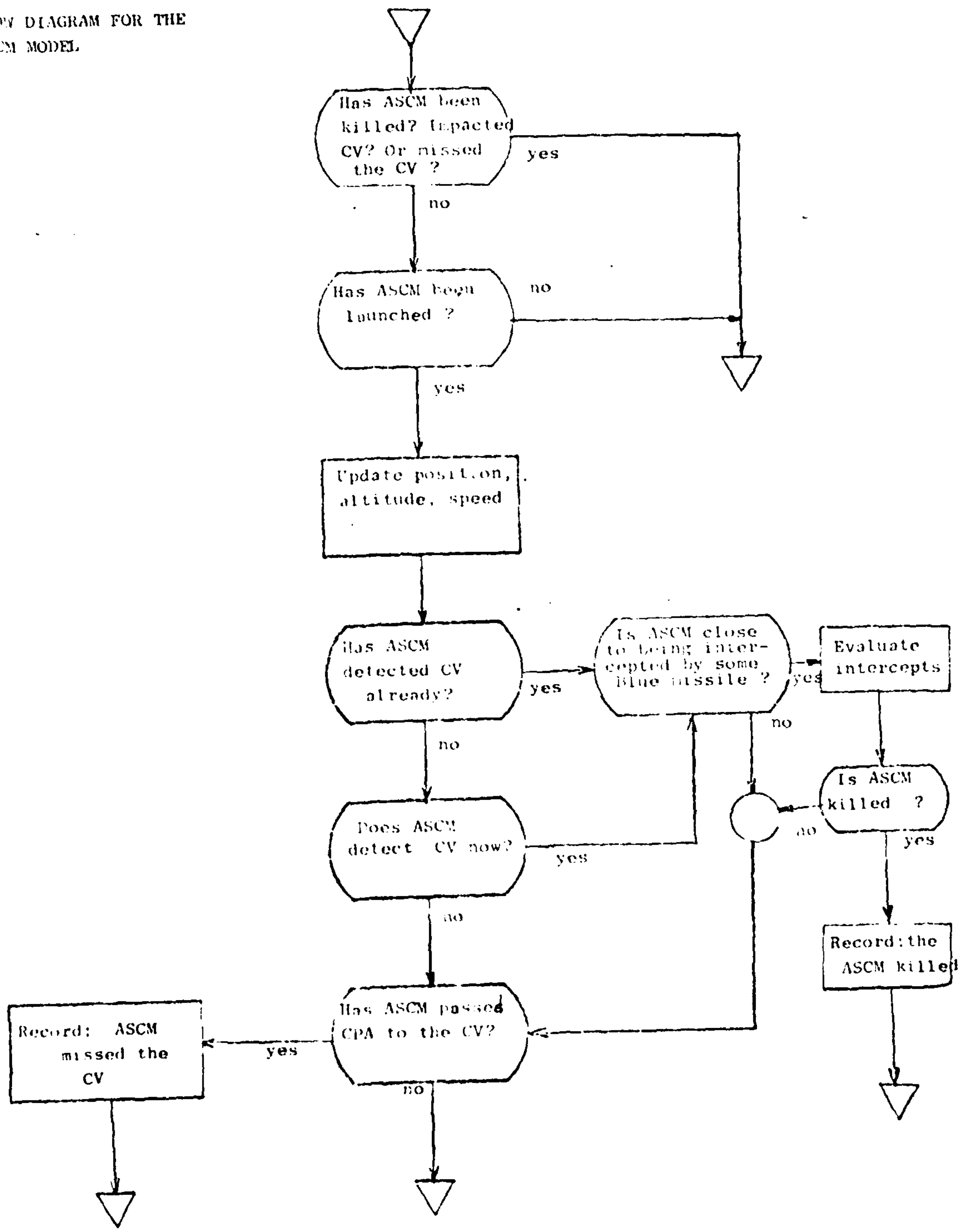


FIGURE A-6
FLOW DIAGRAM FOR THE AEW MODEL

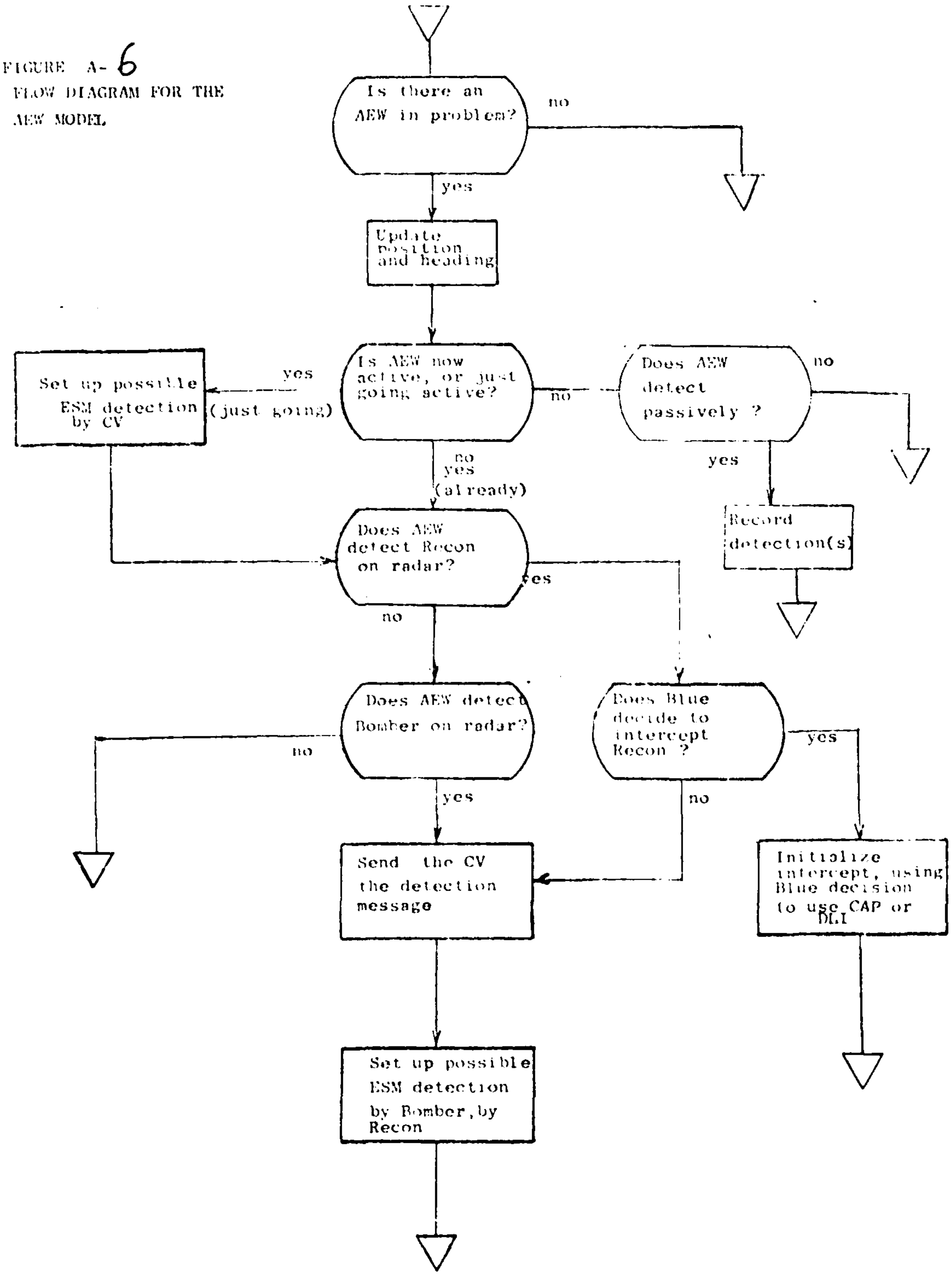


FIGURE A-7
FLOW DIAGRAM FOR AAM
(AIR TO AIR MISSILE)

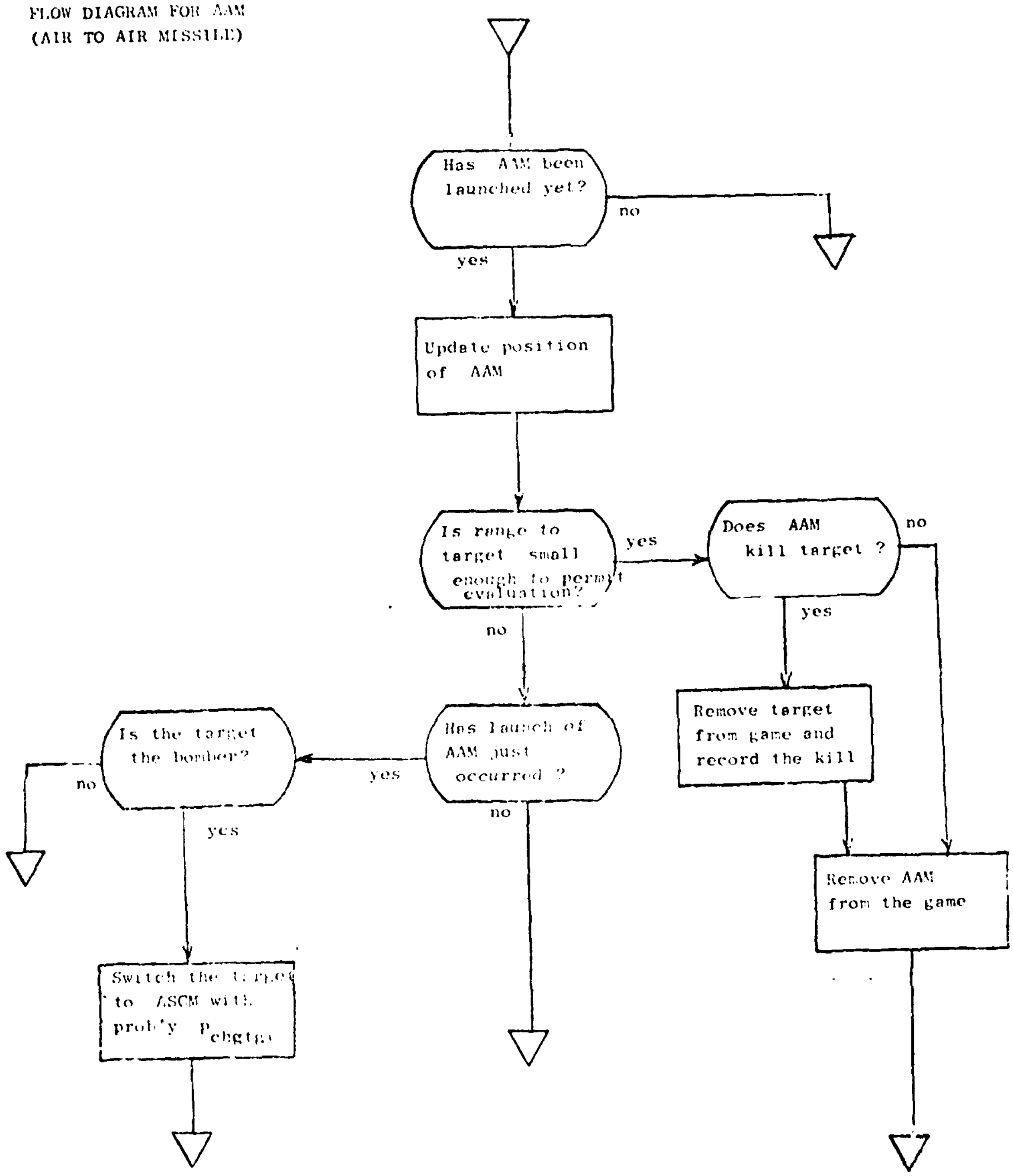


FIGURE A-8

FLOW DIAGRAM FOR THE EXECUTIVE ROUTINE

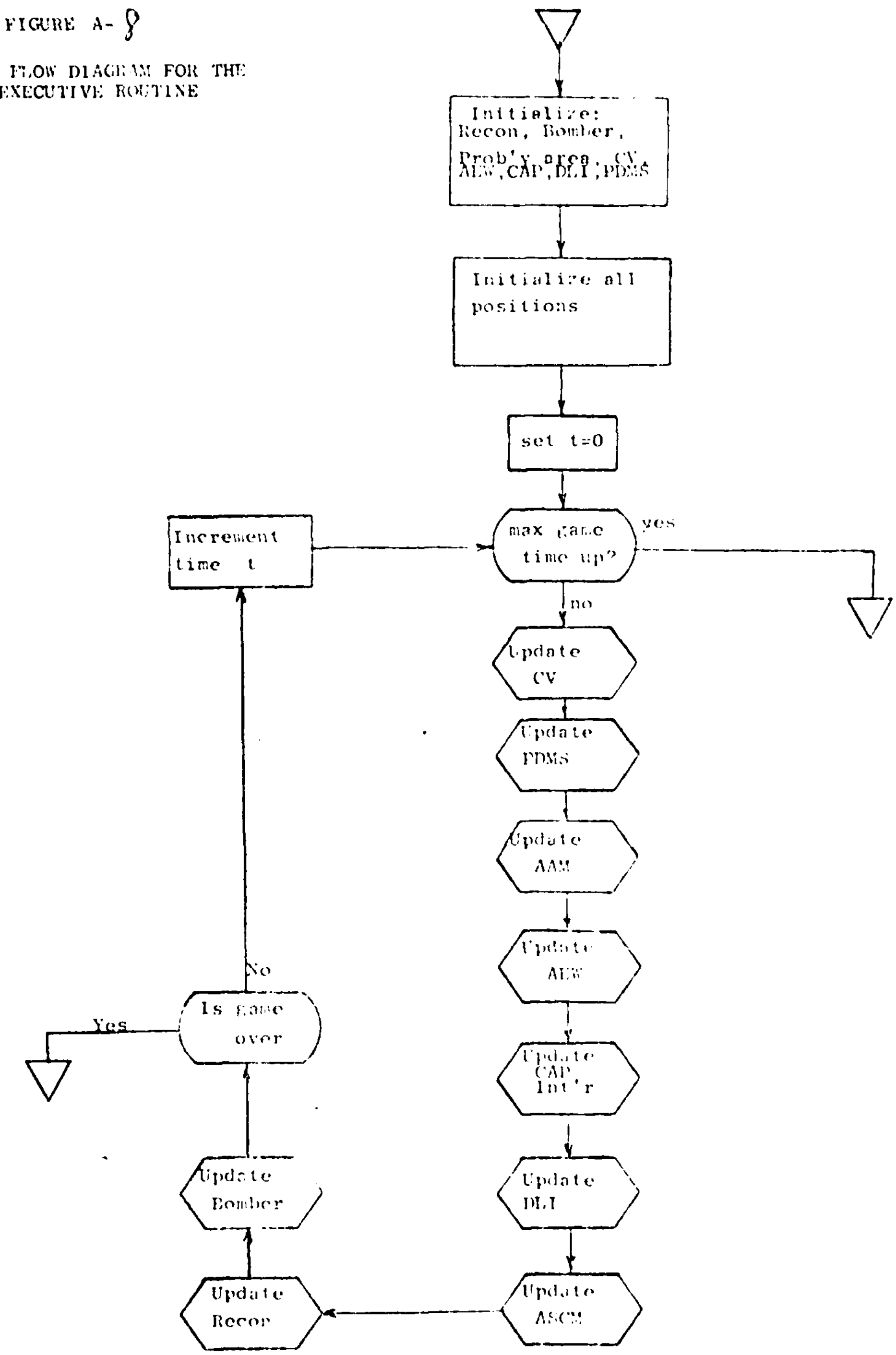
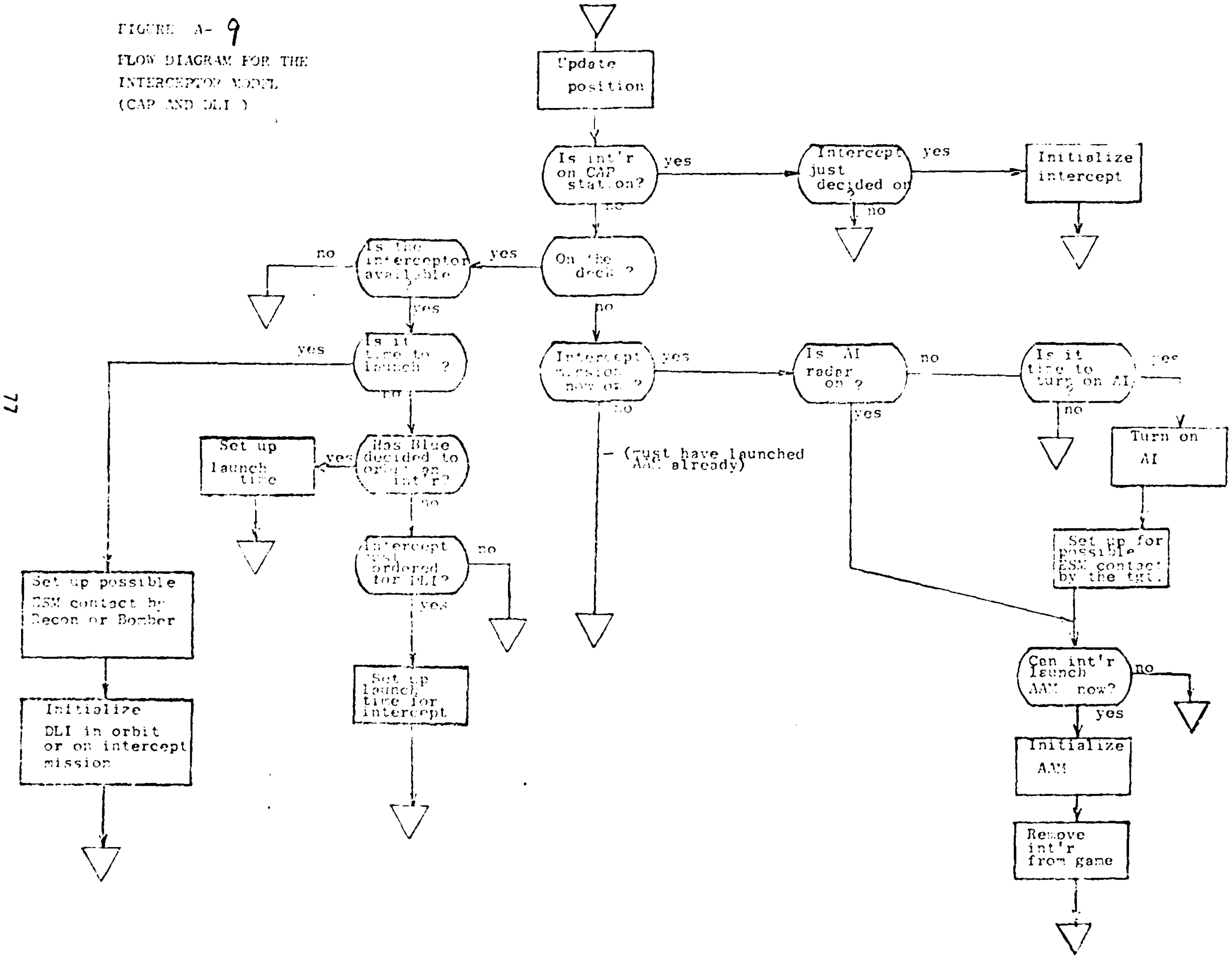


FIGURE A-9
FLOW DIAGRAM FOR THE
INTERCEPTOR MODEL
(CAP AND DLI)



Appendix B

PARAMETERS FOR THE FLEET DEFENSE MODEL

APPENDIX D
PARAMETERS FOR THE MONTE CARLO FLEET DEFENSE MODEL

A. Air-to-Air Missile

1. Search model is cookie-cutter, with ranges tabulated below:

Recon	Bomber	ASCM type A	ASCM type B

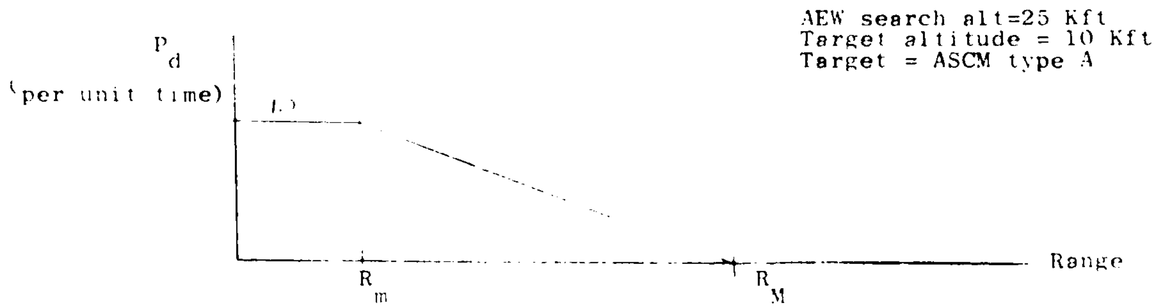
2. Speed of AAM = 3.5 Mach
3. Kill probabilities are a function of launch range and target type, linear interpolation is used on range. Basic values are:

	Recon	Bomber	Type A	Type B
R_m=20 nm	.9	.8	.7	.6
R_M=80 nm	.6	.6	.5	.5

B. AEW Aircraft

4. Speed while on CCAP station = 180 kts
5. Altitude " " " " = 25 Kft
6. Search model for passive mode: line of sight, with probability .4 on discrete signals (messages)
7. Search model for radar:

For an AEW search altitude, specify a probability of detection per unit time vs. range, type of target, and altitude of target. By using a max and min range and linearity the model is of the form illustrated below:



Parameters to be selected are R_m and R_M for time unit of (say) one minute. Choose these by fitting an average detection range and possibly a standard deviation of detection range. Alternatively, one can fit R_m and the average or R_M and the average. An average detection range table follows.

AEW vs.	Recon	Bomber	ASCM A	ASCM B
	475 nm	250	150	200

THIS APPENDIX INCOMPLETE--FURTHER VALUES AVAILABLE IN PROJECT NOTES

Appendix C

EVENT-FLOW DIAGRAM (EFD) FOR FLEET DEFENSE SCENARIO, AND DERIVED AGGREGATED EVENT-STATE TREE

FIGURE C-1
EVENT-FLOW DIAGRAM
FOR FLEET DEFENSE SCENARIO

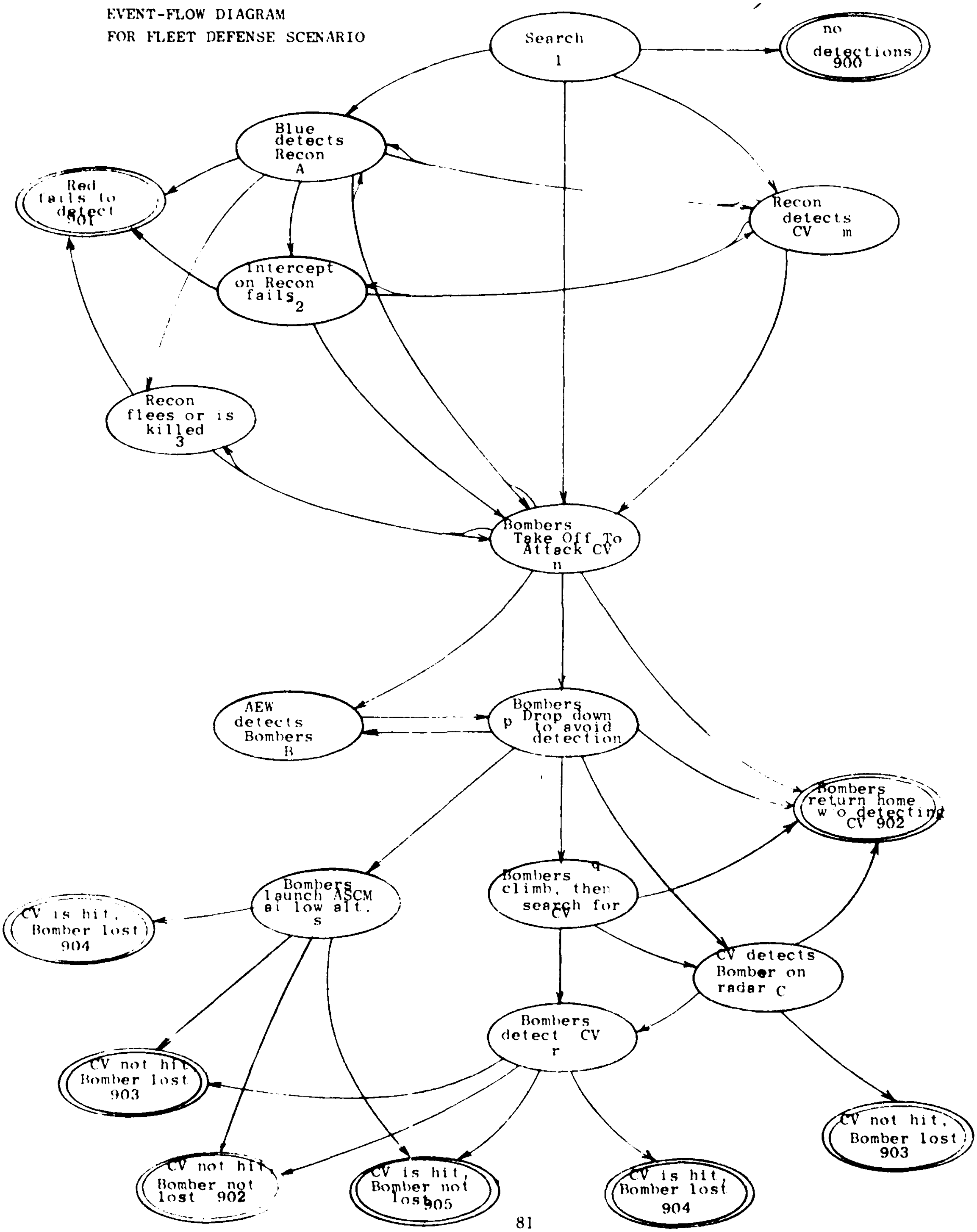
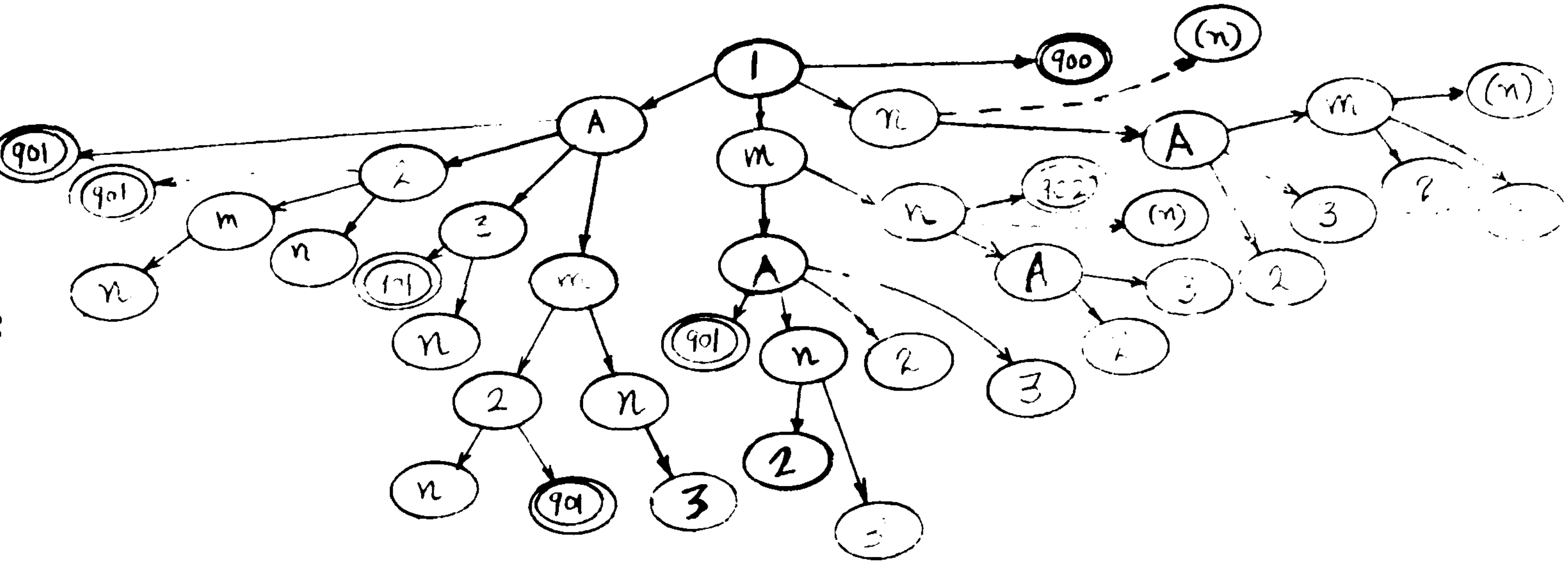


FIGURE C-2

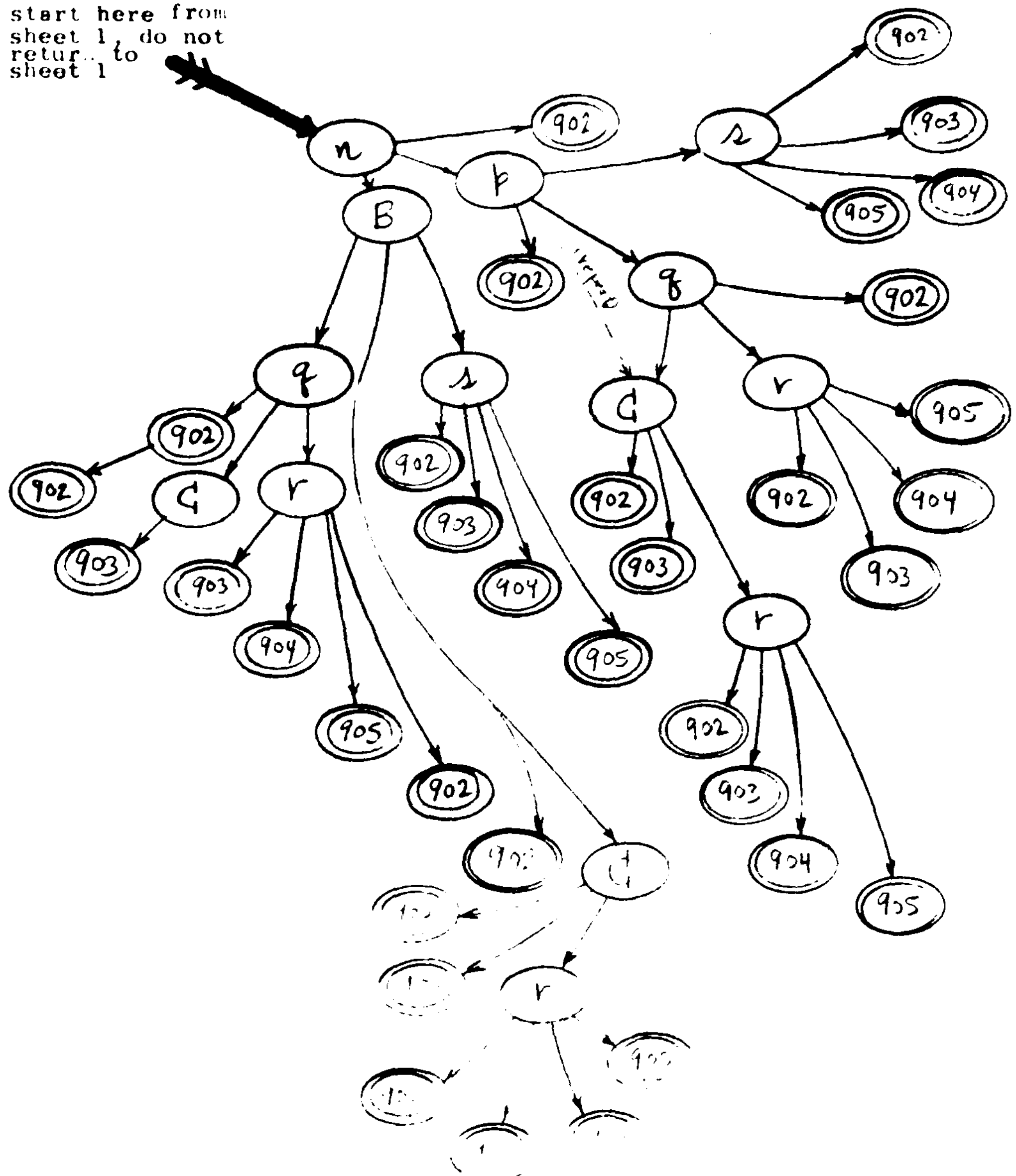
AGGREGATED EVENT STATE TREE
FOR FULL SCENARIO



Remarks: This part of EST is obtained from Event Flow Diagram of Figure by tracing paths until the loops are removed on the top part of the EFD. Each 'end element' here goes to either event "n" or its successors. If (n) is shown, it can be replaced by the entire sheet 2. If (n) is shown it can be replaced by the successors to n on sheet 2. Other states go to successors of n if there is already an n appearing on the path, and to n itself otherwise.

There are six "n", five "n-3", nine "n-2", and three " (n) " elements on this sheet that are non-terminal. Also, there are 32 non-terminal elements on this sheet. Thus the number of non-terminal elements is 32 + (6 + 9 + 3)*(14- 1) = 266, the "14" being from sheet 2.

FIGURE C-2 (cont'd)
AGGREGATED
EVENT STATE TREE FOR BOTTOM OF EFD



Remarks: there are 14 non-terminal and 35 terminal elements on this sheet.

REFERENCES

1. William H. Frye, "An Approach to a Game Theoretic Treatment of Fleet Defense," Technical Note TN-66, Contract N00014-76-C-0633, SRI Project 4840, Stanford Research Institute, Menlo Park, California (30 April 1976)

2. J. C. Harsanyi, "Games With Incomplete Information Played by "Bayesian" Players, Part I: The Basic Model," Management Science, Vol. 14, No. 3 (November 1967)